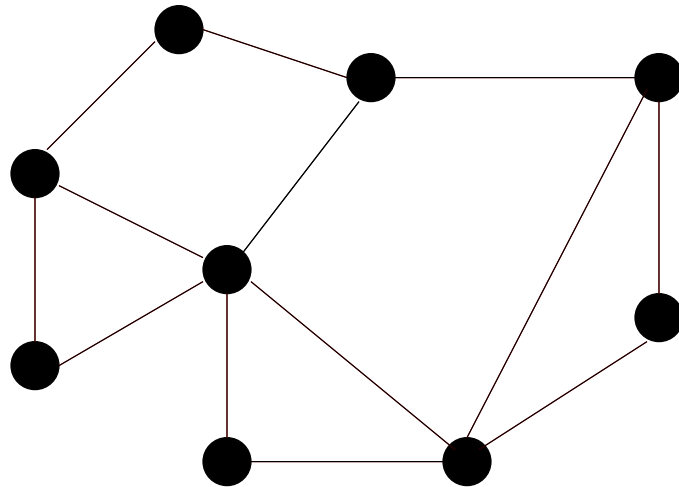


Graphs

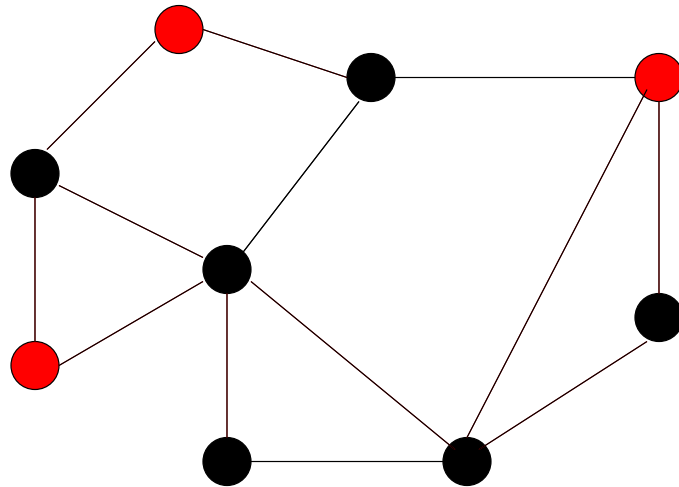
A **graph** is a set of objects, called **vertices**, where some pairs of objects are joined by lines, called **edges**. The vertices are shown as black dots.



Graphs are really useful for modelling the relationships that exist between objects in a network or system.

Independent sets

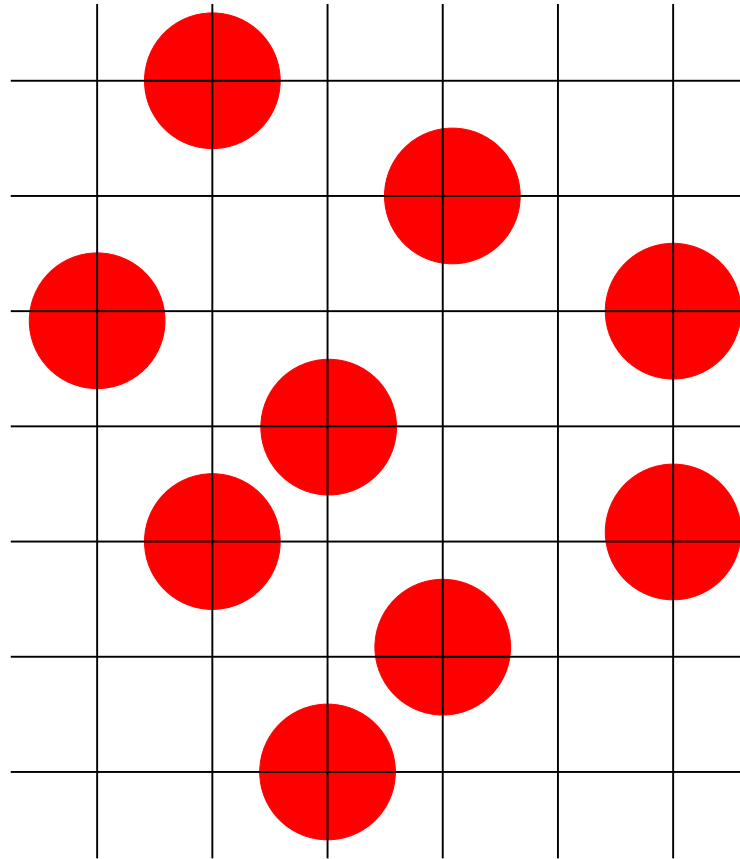
A subset of vertices is an **independent set** for the graph if there is no edge joining two vertices in the independent set. In our picture, the vertices in the independent set are red.



It is easy to find small independent sets. But how big is the **LARGEST** independent set for this graph?

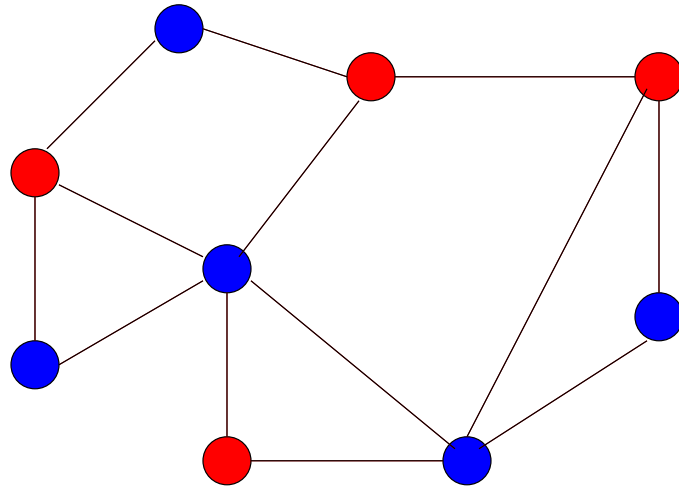
Physicists use independent sets in **grid graphs** to model gas molecules. The idea is that a gas molecule is so large that you cannot have two molecules in adjacent positions on the grid.

The **LARGEST** independent set tells you the biggest number of molecules that you can find in that region of space.



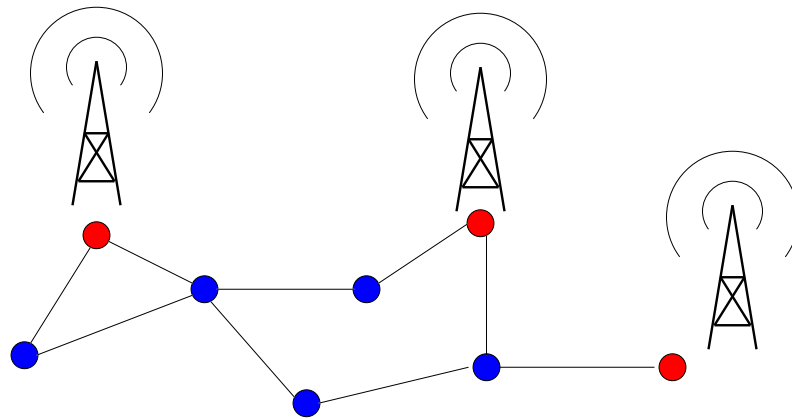
Dominating sets

A subset of vertices is a **dominating set** for the graph if every vertex is either in the dominating set (shown as red), or has a neighbour in the dominating set (shown as blue).



It is easy to find large dominating sets. But how big is the **SMALLEST** dominating set for this graph?

Suppose that you had to build some radio transmitters to cover some given locations. Build a graph where the vertices are the possible **locations** and an edge says that a signal from one location can be picked up by the other one.



For every location to receive a signal, you should build the radio transmitters on a **dominating set** in the graph. But radio transmitters are expensive, so you want to find the **SMALLEST** dominating set you can.