The Department of Applied Mathematics at UNSW is one of the largest and most successful in Australia. It is an active centre for research, both fundamental and applied. Department members have worked on a wide range of research projects in applied mathematics, and have participated in collaborative research activities with numerous distinguished visitors. Department members have received prestigious awards and international recognition for outstanding research in recent years. Research areas of the Applied Department at UNSW cover 5 broad groupings: Biomathematics; Computational Mathematics; Fluid Dynamics, Oceanography and Meteorology; Nonlinear Phenomena; and Optimisation.

Honours thesis topics being offered in 2012 cover all 5 areas. Note that your interests may well be catered for in more than one of the groups below. For example, if your interests are in financial Mathematics, you may find topics of interest in Computational Mathematics, Nonlinear Phenomena and/or Optimisation.

The indicated staff member will be most happy to discuss the possible topics with you further. You can find staff contact details at:

http://www.maths.unsw.edu.au/contact/staff-directory

For any further queries, please feel free to consult the Applied Honours Coordinator:

Dr. William McLean, Room: RC-2085 Phone: 9385 7045.
Biomathematics
Adelle Coster

• **The Insulin Signalling Pathway in Adipocytes — A Mathematical Investigation.** The insulin signalling pathway in adipocytes (fat cells) is the main controller of the uptake of glucose in the cells. Understanding of this system is vital in the investigation of diabetes, which is a deficiency in this control system. One of the experimental techniques used to understand this system is total internal reflection fluorescence (TIRF) microscopy. In this technique, the glucose transporter proteins in the cell have been modified to contain a fluorescent segment. Under the TIRF microscope the first 200nm of the cell is observed, so that the proteins, embedded in small spheres called vesicles, can be detected as they are transported to the surface. These vesicles can then fuse with the cell surface, releasing the glucose transporter proteins. This project will develop a mathematical description of the movement of these vesicles in the TIRF field of view, and also the diffusion of the proteins if the vesicle fuses with the cell surface. This involves the development and solution of differential equations defining the intensity of the resulting image as a function of both space and time, taking into account the TIRF system, the depth of the vesicle and also the relative opacity of the cell interior. This will involve analytical work and possibly some computational simulation of the system. Experimental data for comparison with the model will be available in collaboration with Prof David James at the Garvan Institute for Medical Research.

• **Dynamics and Stability of Cardiac Pacemaker Cells.** The ionic currents underlying the electrical behaviour of pacemaker cells in the heart may be described using coupled nonlinear differential equations. These have been derived in a number of different models from a wide range of electrophysiological data in the literature. In this project, the dynamics of this system, both in single cells, and coupled networks of cells will be explored, and the robustness of the models to changes in parameters ascertained. Different models for the sinoatrial node cells will be assessed and their different responses correlated to the ionic currents, with a view to further optimising the system. This project will be both analytical and computational.
Bruce Henry

- **Pattern formation with anomalous diffusion: How did the leopard get its spots?** This is one of many examples of pattern formation that has been mathematically modelled using reaction-diffusion equations. In this mathematical model the diffusion is treated as standard Brownian motion. In recent years there have been numerous experiments in biological systems showing diffusion that cannot be treated as standard Brownian motion. How can we model this anomalous diffusion mathematically and how can we model pattern formation in these systems?

- **Mathematical modelling of plaque build up in Alzheimer’s disease.** Currently about 150,000 Australians suffer from cognitive impairment due to Alzheimer’s disease and this number is expected to double over the next few decades. One of the features of this disease at the neural level is the accumulation of extracellular amyloid beta plaque but the mechanism for this accumulation and the role that it plays in Alzheimer’s is not yet understood. In this project you will review the literature in this field and you will develop mathematical models for the growth of plaque deposits.

John Murray

- **Statistical analysis of sexually transmitted infections in a longitudinal study in Papua New Guinea** HIV infection is of concern in Papua New Guinea where prevalence is between 1 and 2% of the adult population. Sexually transmitted infections (STI) such as syphilis and herpes simplex virus have been implicated in the increased transmission and acquisition of HIV. Moreover these and other STI are at high levels in PNG. As part of an Australian Aid funded project longitudinal studies have been commenced in two regions in PNG. This project will analyse the data from these projects and investigate the relationships between background factors and incidence of STI.

- **Bioinformatic analysis of HIV founder sequences** There is currently no vaccine for HIV although a large global effort is being conducted to develop one. A National Health and Medical Research Council (NHMRC) funded Program grant involving UNSW and other groups throughout Australia is contributing to this effort. One aspect of this investigation is analysis of the envelope sequence that codes for the surface proteins on HIV that can be recognized by antibodies. Given
the high mutation rate of HIV this part of the sequence changes considerably over the course of infection. An effective vaccine must be structured to respond to the founder virus, in other words the virus that establishes the infection. This project will compare envelope sequences from patients with very early infection to those in later stages in an attempt to delineate the aspects of HIV that might be targeted with a vaccine. The project will use bioinformatic tools available in Matlab, and also employ Operations Research methods.

- **Mathematical modeling of the persistence of HIV infection** Although antiretroviral therapy suppresses HIV infection to low levels, it is not eradicated. There is considerable controversy as to the reasons for this. Under NHMRC funded projects our group has extensively analysed the different components of HIV infection, and also obtained measures of the immune response. This project will involve mathematical modeling that incorporates some of this data in an effort to better understand the interaction of the many processes that contribute to the maintenance of HIV. Identifying which of these are dominant can better direct efforts in the goal of eliminating HIV infection. This project will involve constructing differential equation models, fitting these to data, and some statistical analysis. This will be performed in Matlab.

- **Mathematical modeling of interventions aimed at reducing the spread of HIV in resource poor countries** Resource poor countries have limited means to treat individuals once they have been diagnosed with HIV infection. Although antiretroviral therapy is funded to a large extend by international groups, this therapy still only reaches a fraction of those in AIDS stage of the disease. Prevention is a much cheaper and more effective means of reducing the burden of HIV in a country. Limited health budgets mean however that the prevention methods need to be carefully chosen for maximal impact and at minimal cost. This project will investigate different forms of HIV prevention using mathematical models. The models will be based on work already used in the analysis of HIV prevention measures in Papua New Guinea. Differential equation models and agent-based models may be components of this project.

- **Within host dynamics of emerging drug resistant hepatitis B virus.** Approximately 400 million people world-wide are chronically infected with hepatitis B virus (HBV). Over the long-term it can lead to cirrhosis of the liver and hepatocellular carcinoma. Viral replication can be limited through the use of antiviral drugs but these can fail
with the development of drug resistance. Combination therapy is now being used to combat this type of failure. This project will include mathematical modelling of the development of drug resistant HBV, and analyse the advantages of different schedules of drug combinations.

- **The effects of contact network structure on the spread of diseases.** The transmission of disease within a community is not uniform among individuals. In many instances a small number of “high-transmitters” can greatly influence the resulting epidemic. If the interaction between individuals is modelled by a network, then the high-transmitters can be thought of as nodes with many connections. This project will model transmission over a network with differing degrees of connectivity.

- **Scheduling of cancer chemotherapy with feedback of information during treatment.** Most cancer chemotherapy modelling assumes information is either available completely during treatment, or is known to within some error before treatment but that no new information is obtained after that point. Neither of these assumptions is accurate for clinical treatment of cancer, where there is a large uncertainty in a patient’s response to therapy but that often some information is obtained during treatment that can lead to modification of therapy. This project will use optimization with uncertainty to model clinical practice of cancer chemotherapy scheduling.

**Computational mathematics**

**Bill McLean**

- **Hierarchical Matrices.** In 1999, Wolfgang Hackbusch introduced the concept of a hierarchical matrix or H-matrix. The idea grew out of a computational algorithm for solving integral equations, known as panel clustering. If $A$ is an $N \times N$, dense matrix then the obvious representation of $A$ stores the $N^2$ matrix entries, and computing a matrix-vector product $Ax$ in the obvious way costs $N^2$ operations. However, for a wide class of matrices arising as discretizations of integral and other linear operators, an H-matrix representation uses only $O(N)$ storage and allows the computation of $Ax$ using only $O(N)$ operations, accurate to within the order of the discretization error. It is even possible to develop a whole fast algebra of matrices, with many interesting applications for large-scale numerical simulations. A project in this area would
involve a mix of theory and computation, depending on the background of the student.

Thanh Tran

- **A posteriori error estimates for the finite-element method of lines for nonlinear parabolic equations.** Many time-dependent phenomena can be formulated as a boundary value problem with a nonlinear parabolic equation. When approximation techniques, e.g. the finite-element method of lines, are used to find the approximate solution, there is a need to measure the error. A posteriori error estimates are designed to judge where the computed solution better approximates the exact solution, and where it fails to do so. These error estimates are a fundamental component in the design of reliable and efficient adaptive algorithms for solving the equations. In this project you will learn some of these techniques and do experiments with some practical problems.

- **Interpolation on the sphere by spherical splines.** Data interpolation and fitting problems where the underlying domain is the sphere arise in many areas including, e.g., geodesy and earth science in which the sphere is taken as a model for the earth. In this project you will learn how to use piecewise polynomials on the sphere (which are called spherical splines) to solve these problems. Experiments with scattered data obtained from a NASA satellite will be carried out.

- **Boundary element methods.** Boundary element methods have long been used in engineering to solve boundary value problems. These problems are formulated from many physical phenomena, ranging from mechanical engineering (e.g., in car design) to petroleum engineering (e.g., for simulation of fractured reservoirs). In this project, you will first learn basic concepts of boundary element methods, how to implement and analyse efficiency and accuracy of the methods. Then, depending on your needs and interests, you will use the methods to solve practical problems in engineering or geodesy. Problems in geodesy will involve programming with data collected by a NASA satellite, which may contain up to almost 30 million points.
Fluid dynamics, oceanography and meteorology

Gary Froyland

- Lagrangian coherent structures in ocean and atmosphere models. Recent theoretical advances have enabled the application of matrix-theoretic approaches to study the dynamical transport of mass in ocean and atmosphere models. Advection based transport (as opposed to diffusion based transport) is most strong in so-called “Lagrangian coherent structures” (LCSs). These LCSs are now possible to detect using matrix-theoretic methods. The matrix-theoretic methods are also highly effective at quantifying the water or air mass transported along these coherent pathways. This project will develop efficient numerical approaches for implementing the latest matrix-based approaches and apply them to state-of-the-art ocean and atmosphere models to answer questions concerning the identification of new coherent pathways and the quantification of mass transport. There is a possibility to undertake a joint project with Mark Holzer.

Mark Holzer

- Transport in geophysical fluids: implications for air quality, ocean biogeochemistry, and climate. What fraction of the air you breath had its last surface contact in China? How much of this air passed through the stratosphere? What are the meteorological conditions that lead to intercontinental transport of pollution with minimal dilution? What are the paths of stratospheric ozone through the troposphere in the current and future climate and what are the implications for air quality? How much of the worlds deep water was last ventilated in the Southern Ocean and what is the distribution of times for this water to be re-exposed to the atmosphere? How do oceanic transport and biogeochemistry interact to control the oceans biological productivity? What do measurements of dissolved trace gases tell us about the distribution of times and places where water last equilibrated with the atmosphere (taking up greenhouse gases)? These are some of the key questions that you can tackle using Green-function techniques, modelling, and data analysis to quantify how mass, chemical, and/or thermodynamic properties are transported through the nonlinearily evolving atmosphere and ocean. In your research you can employ a hierarchy of models ranging from simple analytical models, to box models, to global numerical models. A complementary approach
would apply maximum-entropy and other inversion methods to tracer observations to extract the underlying transport operator.

Nonlinear phenomena

Gary Froyland

- **Topics in dynamical systems and ergodic theory.** Ergodic theory is the study of the dynamics of ensembles of points, in contrast to topological dynamics, which focusses on the dynamics of single points. A number of theoretical Honours projects are available in dynamical systems and ergodic theory, aiming at developing new mathematics to analyse the complex behaviour of nonlinear dynamical systems. Depending on your background, these projects may involve mathematics from Ergodic Theory, Functional Analysis, Measure Theory, Nonlinear Time Series Analysis, Nonlinear and Random Dynamical Systems, Markov chains, Graph Theory, and Coding and Information Theory. Successful completion of Math3201 is recommended for this topic.

- **Transfer operator analysis with applications to fluid mixing.** A transfer operator is a linear operator that completely describes the evolution of probability densities of a nonlinear dynamical system. Transfer operators are therefore fundamental objects like discrete time maps and continuous time flows, but operate on ensembles rather than single points. Spectral techniques using transfer operators have recently been shown to be particularly effective for analysing complex dynamics in a variety of theoretical and physical systems, and are an active research area internationally. This project will focus on developing powerful transfer operator techniques to extract important geometric and probabilistic dynamical structures from uid-like models. If desired, application areas include the ocean (an incompressible uid) and the atmosphere (a compressible uid). Successful completion of Math3201 is recommended for this topic.

Bruce Henry

- **Fractional calculus for fractals.** A fractal function such as the Mandelbrot–Weierstrass function is everywhere continuous but nowhere differentiable. However this function is fractionally differentiable. In this topic you will learn about fractional calculus and to what extent fractional calculus provides a calculus for fractals.
Random walks on discrete lattices. A grasshopper jumps from one brick to another on a brick wall until it reaches a brick at the edge and then it is promptly squashed. If it starts in the middle of a brick wall that is one hundred bricks high what is the probability that it is squashed at the edge brick that is eleventh from the top? This is an example of a random walk problem on the triangular lattice. In this project you will be solving the random walk problem on other lattices.

Statistical mechanics of small particle systems. The fundamental postulate of equilibrium statistical mechanics is that in an isolated system all parts of the energy surface are equally probable states. But the phase space trajectories for most isolated systems with just a few particles do not visit all regions of their energy surface with equal frequency. They hang around in some regions for a long time and hardly ever visit other regions. In this project you will use perturbation methods to find the regions that are visited most frequently and you will then use this information to determine statistical mechanical properties of small systems.

John Roberts

Algebraic dynamics The topic is broadly taken to be the intersection of algebra, number theory, and dynamical systems. This interdisciplinary area of research is cutting edge and exciting, and has important applications to, e.g., cryptography, random matrix theory, materials science and engineering.

Discrete integrable systems. The study of integrable (partial) difference equations and integrable maps is presently a very active field of research. In the first instance, this is due to the increasingly numerous areas of physics in which such systems feature. The study of discrete integrable systems also has intrinsic mathematical appeal, broadly speaking to do with finding analogues of concepts or properties (e.g., the Painleve property, Lax pairs, Hamiltonian structure) that exist in integrable systems with continuous time.

Wolfgang Schief

Topics in soliton theory. Solitons constitute essentially localised nonlinear waves with remarkable novel interaction properties. The soliton represents one of the most intriguing of phenomena in modern
physics and occurs in such diverse areas as nonlinear optics and relativity theory, plasma and solid state physics, as well as hydrodynamics. It has proven to have important technological applications in optical fibre communication systems and Josephson junction superconducting devices. Nonlinear equations which describe solitonic phenomena (“soliton equations” or “integrable systems”) are ubiquitous and of great mathematical interest. They are privileged in that they are amenable to a variety of solution generation techniques. Thus, in particular, they generically admit invariance under symmetry transformations known as B"acklund transformations and have associated nonlinear superposition principles (permutability theorems) whereby analytic expressions descriptive of multi-soliton interaction may be constructed. Integrable systems appear in a variety of guises such as ordinary and partial differential equations, difference and differential-difference equations, cellular automata and convergence acceleration algorithms. It is by now well established that there exist deep and far-reaching connections between integrable systems and classical differential geometry. For instance, the interaction properties of solitons observed in 1953 by Seeger, Donth and Kochend"orfer in the context of Frenkel and Kontorovas dislocation theory, and later rediscovered by Zabusky and Kruskal (1965) in connection with the numerical treatment of the important Fermi–Pasta–Ulam problem, are encoded in the geometry of particular classes of surfaces governed by the sine-Gordon equation and Korteweg-de Vries (KdV) equation respectively. The geometric study of integrable systems has proven to be very profitable to both soliton theory and differential geometry. Integrable systems play an important role in discrete differential geometry. The term “discrete differential geometry” reflects the interaction of differential geometry (of curves, surfaces or, in general, manifolds) and discrete geometry (of, for instance, polytopes and simplicial complexes). This relatively new and active research area is located between pure and applied mathematics and is concerned with a variety of problems in such disciplines as mathematics, physics, computer science and even architectural modelling. Specifically, theoretical and applied areas such as differential, discrete and algebraic geometry, variational calculus, approximation theory, computational geometry, computer graphics, geometric processing and the theory of elasticity should be mentioned. Soliton theory constitutes a rich source of Honours topics which range from applied to pure. Specific topics will be tailored towards the preferences, skills and knowledge of any individual student.
• **A deeper understanding of discrete and continuous systems through analysis on time scales.** Historically, two of the most important types of mathematical equations that have been used to mathematically describe various dynamic processes are: differential and integral equations; and difference and summation equations, which model phenomena, respectively: in continuous time; or in discrete time. Traditionally, researchers have used either differential and integral equations or difference and summation equations — but not a combination of the two areas — to describe dynamic models. However, it is now becoming apparent that certain phenomena do not involve solely continuous aspects or solely discrete aspects. Rather, they feature elements of both the continuous and the discrete. These types of hybrid processes are seen, for example, in population dynamics where nonoverlapping generations occur. Furthermore, neither difference equations nor differential equations give a good description of most population growth. To effectively treat hybrid dynamical systems, a more modern and flexible mathematical framework is needed to accurately model continuous–discrete processes in a mutually consistent manner. An emerging area that has the potential to effectively manage the above situations is the field of dynamic equations on time scales. Created by Hilger in 1990, this new and compelling area of mathematics is more general and versatile than the traditional theories of differential and difference equations, and appears to be the way forward in the quest for accurate and exible mathematical models. In fact, the field of dynamic equations on time scales contains and extends the classical theory of differential, difference, integral and summation equations as special cases. This project will perform an analysis of dynamic equations on time scales. It will uncover important qualitative and quantitative information about solutions; and the modeling possibilities. Students who undertake this project will be very well equipped to make contributions to this area of research.

• **Advanced Studies in differential equations and nonlinear analysis** Many problems in nonlinear analysis can reduced to the study of the set of solutions of an equation of the form $f(x) = p$ in an appropriate space. This project will give the student an introduction to nonlinear analysis and its applications. A student who completes this project will be well-prepared to make the transition to research studies in related fields.
• **Advanced studies in nonlinear difference equations.** Difference equations are of huge importance in modelling discrete phenomena and their solutions can possess a richer structure than those of analogous differential equations. This project will involve an investigation of nonlinear difference equations and the properties of their solutions (existence, multiplicity, boundedness, etc). Students who complete this project will be very well-equipped to contribute to the research field.

**Optimisation**

Gary Froyland

• **Topics in integer programming and combinatorial optimisation.** Integer programming is a mathematical framework for solving large decision problems. Usually there is some underlying discrete structure for the problem such as a network or graph. You will learn new mathematical techniques in discrete mathematics, algebra, and geometry. If desired, application areas may include scheduling airlines, rail, or mining processes. Successful completion of Math2140 or Math3041 is required for this project.

• **Stochastic integer programming.** Almost all real world models have significant uncertainty in their measured data. A naive approach is to replace probability distributions of data with their mean value and create a single deterministic model. However, optimising this deterministic model typically results in decisions that are far from optimal. In order to make better decisions, the underlying probability distributions must be properly incorporated into the optimisation process. This is the aim of stochastic programming. The aim of this project is to develop rigorous optimization methods that include uncertainties in the forecast data and evaluate all possible options in light of the latest information. Successful completion of Math2140 or Math3041 is required for this project. Familiarity with probability theory is essential. If desired, application areas may include scheduling airlines, rail, traffic, or mining processes.

• **Polyhedral analysis of fundamental problems.** Understanding the polyhedral structure of the feasible set of solutions to an optimization problem is a key component of solving the problem. The aim of this project is to undertake a polyhedral analysis of fundamental problems that are building blocks of larger problems. For example, the precedence constrained knapsack problem (PCKP), in which items are
Vaithilingam Jeyakumar

- **Multi-task supervised learning by convex optimization.** Most approaches to (single task) supervised learning and data mining intrinsically involve optimization. Nowadays, optimization techniques are applied in a great variety of practical machine learning and data mining problems. In many practical situations a number of statistical models need to be estimated from data. For example, in finance forecasting models for predicting the value of many possibly related indicators simultaneously is often required; in marketing modelling the preferences of many individuals simultaneously is common practice. Recent research has shown that learning multiple related tasks from data simultaneously can be advantageous in terms of predictive performance relative to learning these tasks independently. The aim of the project is to examine and develop an approach to multi-task learning based on convex optimization similar to existing ones, such as the one for Support Vector Machines, that have been successfully used (and examined in three successful honours theses) in the past for single-task learning.

- **Robust optimization and data mining.** In many real world problems, the data associated with the underlying optimization problem are often uncertain due to modelling errors. Various techniques, such as stochastic programming and scenario optimization, have been developed to address these optimization problems under uncertainty. Robust optimization, which is based on a description of uncertainty by sets, instead of probability distributions, is emerging as a powerful methodology to examine uncertain optimization problems. This project will examine various robust optimization approaches to solving uncertain optimization problems. Application areas may include data mining and machine learning.

- **Semi-algebraic geometry and polynomial optimization.** What has algebraic geometry to do with optimization? The answer is: quite a lot. And all this is due to innovative ideas and links discovered in the last decade between pure and applied mathematics. A good
understanding of convex sets in algebraic geometry will lead to insights into solving hard optimization problems involving polynomials. This project will examine emerging applications of algebraic geometry to optimization over polynomials.

John Murray

- **Scheduling of cancer chemotherapy with feedback of information during treatment.** Most cancer chemotherapy modelling assumes information is either available completely during treatment, or is known to within some error before treatment but that no new information is obtained after that point. Neither of these assumptions is accurate for clinical treatment of cancer, where there is a large uncertainty in a patients response to therapy but that often some information is obtained during treatment that can lead to modification of therapy. This project will use optimization with uncertainty to model clinical practice of cancer chemotherapy scheduling.

Chris Tisdell

- **Exact controllability of dynamical systems and their applications.** Control of physical systems are of high importance in economic, biological and industrial applications. For example: controlling a physical object through impacts, called impulsive manipulation, arises in a number of robotic applications; optimal harvesting policy for an ecosystem with impulsive harvest is of important ecological value. The aim of those researching problems from control theory is to steer certain systems from the initial state to a desired final state in a finite time. In this project we will analyse the mathematics of control and then prove the existence of a control that guarantees a solution to certain problems is steered from the initial state to a desired final state in a finite time. The methods involved will include progressive elements of applicable analysis and differential equations. Students who undertake this project will be very well equipped to make contributions to this area of research.

Rob Womersley

- **Optimal placement of points on manifolds, for examples spheres and tori.** It is easy to equally distribute points on a circle, but when you move to the sphere, even in just three dimensions, equal distribution becomes impossible except for a few specialized cases (the platonic
solids). This leads to many different criteria: minimum energy (associated with electron charges), covering, packing, interpolation, cubature, voronoi cells etc. Then there are more complicated manifolds such as the torus. This combines aspects of optimization, geometry, classical polynomials, visualization, high-performance computing.

- **Estimation of constrained probability density functions.** The market data for various financial instruments (e.g., call options) contains information about the implied distribution of the underlying asset. This is an under-determined (inverse) problem, and some principle is needed to regularise the problem. Examples are maximizing the entropy of the distribution or minimising the cross entropy or another measure of the distance of the density from another (prior) distribution. Critical to solving these problems is the use of duality to turn an infinite dimensional problem into a finite dimensional one.

- **Constrained covariance matrix estimation using semi-definite programming.** Covariance matrices are required in many areas to quantify the variation in a random variable. Mathematically they are just symmetric positive semi-definite matrices. The interest is in finding covariance matrices which satisfy some linear constraints and also minimize some criteria, for example being closest to a historical covariance matrix. This depends on the relatively new area of semi-definite optimization, in which the variables are symmetric positive semi-definite matrices.

- **Computational methods in finance.** Many financial problems require sophisticated mathematical models, which in turn can only be solved by computational techniques. Reliability and efficiency of the techniques are crucial, as well as the ability to get sensitivity information for risk management purposes. Examples include using QMC methods for multi-asset path dependent financial derivatives and the associated Greeks or getting reliable implied volatility information.