UNSW 51st ANNUAL SCHOOL MATHEMATICS COMPETITION – JUNIOR DIVISION

1. The infinite nested radical

$$c = \sqrt{1 + 2\sqrt{1 + 2\sqrt{1 + 2\sqrt{1 + 2\sqrt{\ldots}}}}}$$

converges. Find c.

2. In how many ways can 10000 be written as a sum of consecutive odd positive integers?

3. Consider the following array of integers

Row 0					1				
Row 1				1	1	1			
Row 2			1	2	3	2	1		
Row 3		1	3	6	7	6	3	1	
Row 4	1	4	10	16	19	16	10	4	1

in which every number is the sum of the number n directly above and the numbers one to the left and one to the right of n. A blank space indicates the number zero. Thus 16=3+6+7.

   (a) Prove that the sum of entries in row k is 3^k.

   (b) Prove that there is at least one even number in each row beyond row 1.

   (c) Prove that the third (non-zero) number from the left in row k is given by $\frac{1}{2}k(k+1)$.

4. Given any two people we may classify them as friends, enemies or strangers. Prove that in a gathering of seventeen people there must be either three mutual friends or three mutual enemies or three mutual strangers.

5. A person of height 1.7 metres leaves a tall building at ground level and walks in a straight line direction up a constant gradient path. They walk under a tall billboard after twenty metres and continue walking up the path for another five metres at which point they turn around and notice that the top of the billboard aligns horizontally with the top of the building. They continue along up the path a further ten metres where they turn around again and notice that the top half of the building is now visible above the billboard. The height of the building is much greater than the height of the billboard which is much greater than the height of the person. What is the height of the building?

6. A travelling sales person tours towns A, B, C, D, E and stays overnight in one of the towns. If they stay overnight in town A then the next night they stay in town B. If they stay overnight in town B then the next night they stay in town C. If they stay overnight in town C then the next night they stay in town D. If they stay overnight in town D then the next night they stay in town E. If they stay overnight in town E they roll two fair dice to determine whether they will return to D for the next night or move on to town A for the next night. They then continue their tour either from D to E or from A to B, etc. What is the long term probability of finding them in town E on any given night in each of the scenarios below:

   Scenario 1

   They return from E to D if the roll of the dice adds up to a number divisible by two, otherwise they move on from E to A.

   Scenario 2

   They return from E to D if the roll of the dice adds up to a number divisible by three, otherwise they move on from E to A.