MATH3121

MATHEMATICAL METHODS & PARTIAL DIFFERENTIAL EQUATIONS

Semester 2, 2015
MATH 3121 - Course outline

Information about the course

Course Authority: Dr. Q. T. Le Gia

Lecturer: Dr. Q. T. Le Gia
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Tutor: Dr. Q. T. Le Gia

Consultation times: Wed 2-4pm or by appointment

Credit, Prerequisites, Exclusion:

- 6 Units of Credit (6UOC)
- Pre-requisites: 12 units of credit in Level 2 Mathematics courses including MATH2011 or MATH2111, and MATH2120 or MATH2130. Note: MATH2520 (alternatively MATH2620) is recommended.
- There is no higher version of this subject.

Location and times: there are 3 lectures, 1 class tutorial per week. Tutorials commence in Week 2 and run until Week 13.

Lectures (weeks 1-12)
   Wednesday  09:00-11:00  Webster Theatre B
   Thursday  13:00-14:00  Old Main Building OMB-145

Tutorials: (weeks 2-13)
   Monday  14:00-15:00  Webster 302
   Or Tuesday  09:00-10:00  Webster 302

Moodle

Further information, skeleton lecture notes, and other material will be provided via Moodle (http://moodle.telt.unsw.edu.au).
Course aims

This course builds on MATH2120 Mathematical Methods for Differential Equations in that it is concerned with ways of solving the (usually partial) differential equations that arise mainly in physical, biological and engineering applications. Analytical methods have considerable intrinsic interest, but their importance for applications is the driving motive behind this course. The main analytical tools developed in this course can be thought of as generalisations of the Fourier and power series representations of functions studied in MATH2120. This leads to new types of functions and to practical methods for solving differential equations. We will pay special attention to functions defined on infinite domains.

The course begins by characterising different partial differential equations (PDEs), and exploring similarity solutions and the method of characteristics to solve them. The Fourier transform, the natural extension of a Fourier series expansion is then investigated. For functions of time, the Fourier transform corresponds to the “spectrum” of the function or signal in the problem in the frequency domain. Closely related to the Fourier transform is the Laplace transform which is particularly useful for solving the initial value PDEs that arise in many physical applications. Although contour integration is an intrinsic part of using these transforms, only brief references to complex variable methods will be made.

Transforms give a wide insight into the behaviour of a function and suggests other possibilities for the integral representation of solutions of PDEs. By exploiting certain special solutions of a given linear PDE we eventually obtain the idea of a Green’s function for the PDE and a corresponding integral form for the solution. The power of Green’s functions can be observed in their use as the inverses of differential operators on both infinite and bounded domains.

Frequently it is not possible to evaluate in closed form the Fourier, Laplace or Green’s function integrals appearing in the solution of the given PDE. All is not lost as we can still explore the asymptotic behaviour of these integrals at large parameter values and obtain physically useful information on the solution of the underlying problem.

Relation to other mathematics course

Mathematics may be divided into the broad categories of analysis (calculus), algebra, geometry and logic.

This subject fits largely into the calculus category and follows on from material you will have learned in first year and from other related courses you may have taken, although algebra and areas will also be involved.

This course is very useful for those majoring in Applied Mathematics, those planning to teach, or those students of Mathematics who are interested in the application of mathematical techniques to real-world problem solving.
Student Learning Outcomes

Students taking this course will develop an appreciation of various techniques, transforms and mathematical methods related to the solution of differential equations. The ability to provide logical and coherent derivations of results, and the ability to solve differential problems via the methods will be paramount.

Through regularly attending lectures and applying themselves in tutorial exercises, students will develop competency in mathematical presentation, written and verbal skills.

Relation to graduate attributes

The above outcomes are related to the development of the Science Faculty Graduate Attributes, in particular: 1. Research, inquiry and analytical thinking abilities, 4. Communication, 6. Information literacy

Teaching strategies underpinning the course

New ideas and skills are introduced and demonstrated in lectures, then students develop these skills by applying them to specific tasks in tutorials and assessments.

Rationale for learning and teaching strategies

We believe that effective learning is best supported by a climate of enquiry, in which students are actively engaged in the learning process. To ensure effective learning, students should participate in class as outlined below.

We believe that effective learning is achieved when students attend all classes, have prepared effectively for classes by reading through previous lecture notes, in the case of lectures, and, in the case of tutorials, by having made a serious attempt at doing for themselves the tutorial problems prior to the tutorials.

Furthermore, lectures should be viewed by the student as an opportunity to learn, rather than just copy down lecture notes.

Effective learning is achieved when students have a genuine interest in the subject and make a serious effort to master the basic material.

The art of logically setting out mathematics is best learned by watching an expert and paying particular attention to detail. This skill is best learned by regularly attending classes.

Class tests and assignments are mainly to provide feedback on a student’s progress and help students to identify problems early. Assessment in this course will use problem-solving tasks of a similar form to those practiced in tutorials, to encourage
the development of the core skills underpinning this course and the development of analytical thinking.

**Assessment**

**Knowledge and abilities assessed:** All assessment tasks will assess the learning outcomes outlined above, specifically, the ability to provide logical and coherent mathematical derivations, and the ability to solve boundary and initial value problems using the methods presented.

**Assessment criteria:** The main criteria for marking all assessment tasks will be clear and logical presentation of correct solutions.

**Assignments**

**Rationale:** Assignments will give an opportunity for students to try their hand at more difficult problems requiring more than one line of argument and also introduce them to aspects of the subject which are not explicitly covered in lectures.

Assignments must be YOUR OWN WORK, or severe penalties will be incurred.

You should consult the University web page on plagiarism [www.lc.unsw.edu.au/plagiarism](http://www.lc.unsw.edu.au/plagiarism)

<table>
<thead>
<tr>
<th>Task</th>
<th>Date</th>
<th>Form of Submission</th>
<th>Weighting</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tutorial submissions*</td>
<td>3 submissions during the semester</td>
<td>Written</td>
<td>6%</td>
</tr>
<tr>
<td>Class test</td>
<td>Week 7</td>
<td>Written</td>
<td>14%</td>
</tr>
<tr>
<td>Short assignments 1,2,3</td>
<td>Due Thurs 13:00, Weeks 5, 9, 12. Late assignments will not be accepted.</td>
<td>Written</td>
<td>15%</td>
</tr>
<tr>
<td>Final Examination (2 hours)</td>
<td>The final examination will assess student mastery of all the material covered in the lectures.</td>
<td>Written</td>
<td>65%</td>
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*Tutorial submissions:* Each tutorial submission will consist of a solution to one tutorial problem, which will be specified during the semester. Students can submit the solutions in hand-written form or by using Moodle wiki. Note that, 3 hand-written submissions will be equivalent to 1 Moodle wiki submission. The wiki submission must not duplicate the solutions which have been available there).

**Additional resources and support**

**Tutorial Exercises**

A set of tutorial exercises will be given out. These problems are for YOU to do to
enhance your mastery of the course.

SOME of the problems will be done in tutorials, but you will learn a lot more if you try to do them before the tutorial.

**Lecture notes**

A set of skeleton notes containing only definitions, theorems and proofs will be provided on Moodle.

**Textbooks**

There is no set text for this course. The content of the course will be defined by the lectures.

**UNSW Moodle**

The School of Mathematics and Statistics makes extensive use of the centrally provided electronic learning environment known as “UNSW Moodle”. Access to this server is via any suitably configured web browser from any computer with an internet connection. The URL for UNSW Moodle is

http://moodle.telt.unsw.edu.au

The School of Mathematics and Statistics web pages for Current Students also have a quick link to UNSW Moodle. For UNSW Moodle your “Username” is z immediately followed by your student number, and your “Password” is your zPass. The Moodle login page has information on how to activate your zPass in order to login and various warnings about difficulties you may encounter with your zPass. There is also a link to the IDM Self Service page where you can reset or “unlock” your zPass if needed. Help for using Moodle is available via links from the UNSW Moodle login page or directly via the URL

https://student.unsw.edu.au/moodle

Once logged in to the UNSW Moodle you will have a choice of modules for all your courses including MATH3121. Outline lecture notes for this course will be made available in this module. Note that these notes are not a substitute for attendance at lectures and tutorials.

**Outline of Syllabus**

The course will include material from the following. Outline lecture notes for this course will be made available via the Moodle web site. They are not a substitute for attendance at lectures and tutorials. The course content is ultimately defined by the material covered in lectures.
Properties of and fundamental methods for the solution of partial differential equations (PDEs); classification of first and second order PDEs; Dirichlet and Neumann problems including the Wave, Heat and Laplace Equations; Dimensionless Similarity Solutions; Method of Characteristics; Fourier and Laplace Transforms; Asymptotic Solutions for Integrals and Green’s Functions.

1. Introduction and revision (week 1)

Classification of 1st and 2nd order PDEs and boundary conditions; well-posedness/superposition; dimensionless variables; similarity solutions; revision of Fourier’s method (separation of variables)

2. The Sturm-Liouville equations (week 2)

Definition of Sturm-Liouville problem; singular Sturm-Liouville problem; orthogonal functions; Legendre’s equation; Bessel’s equation

3. Generalised Fourier series and their applications to PDEs (weeks 3-4)

Fourier-Legendre series; Fourier-Bessel series; the heat equation; the wave equation

4. First order equations: the method of characteristics (weeks 5-6)

Linear and quasi-linear equations; the method of characteristics

5. Laplace transforms (weeks 7-8)

Definition and properties of Laplace transforms; inverse Laplace transforms; solutions of sets of ODEs and PDEs using Laplace transforms

6. Fourier transforms (week 9)

Definition and properties of Fourier transforms; inverse Fourier transforms; solutions of sets of ODEs and PDEs using Laplace transforms

7. Green’s function (weeks 10-11)

Green’s functions for Laplace’s equation and the wave equations; Green’s identities

8. Asymptotic methods for integrals (week 12)

Laplace’s method and the method of stationary case

Additional References
• G. Stephenson, *An Introduction to Partial Differential Equations for Science Students*, (Longman)
• F. B. Hilderbrand, *Advanced Calculus for Applications*, (Prentice-Hall)

Students seeking resources can also obtain assistance from the UNSW Library. One starting point for assistance is: [www.library.unsw.edu.au](http://www.library.unsw.edu.au)

**Course Evaluation and Development**

The School of Mathematics and Statistics evaluates each course each time it is run. We carefully consider the student responses and their implications for course development. It is common practice to discuss informally with students how the course and their mastery of it are progressing.

**Administrative matters**

**School Rules and Regulations**

The School of Mathematics and Statistics has adopted a number of policies relating to enrolment, attendance, assessment, plagiarism, cheating, special consideration etc. These are in addition to the policies of the University. Please take the time to read the following policies.

Individual courses may also adopt other policies in addition to or replacing some of the School ones. These will be clearly notified in the Course Initial Handout and on the Course Home Page on the School of Mathematics and Statistics web site.


Students in courses run by the School of Mathematics and Statistics should be aware of the School and Course policies by reading the appropriate pages School of Mathematics and Statistics web site.


The School of Mathematics and Statistics will assume that all its students have read and understood the School policies on the above pages and any individual course
policies on the Course Initial Handout and Course Home Page. Lack of knowledge about a policy will not be an excuse for failing to follow the procedures in it.

**Plagiarism and academic honesty**
Plagiarism is the presentation of the thoughts or work of another as one’s own. Issues you must be aware of regarding plagiarism and the university’s policies on academic honesty and plagiarism can be found at
http://www.lc.unsw.edu.au/plagiarism
and