MATH3161/MATH5165 Optimization

Lecturers

- A./Prof. Rob Womersley, RC-3062, Ph: 9385-7043, Email: r.womersley@unsw.edu.au (Weeks 1–6, course authority)
- Dr. Guoyin Li, RC-2082, Ph: 9385-7095, Email: g.li@unsw.edu.au (Weeks 7–12)

Course details

- Units of Credit 6
- Prerequisites 12 units of credit in Level 2 Mathematics courses including MATH2011 or MATH2111 or MATH2510, and MATH2501 or MATH2601, or both MATH2019(DN) and MATH2089, or both MATH2069(CR) and MATH2099.
- Exclusions MATH3181

Lectures

The times and locations of the lectures are

<table>
<thead>
<tr>
<th>Day</th>
<th>Time</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday</td>
<td>14:00</td>
<td>CLB 5</td>
</tr>
<tr>
<td>Monday</td>
<td>15:00</td>
<td>CLB 5</td>
</tr>
<tr>
<td>Tuesday</td>
<td>9:00</td>
<td>ChemSci M17</td>
</tr>
</tbody>
</table>

The lectures will be common to both MATH3161 and MATH5165 students. Students in the graduate version (MATH5165) are expected to display much more independence, working through all tutorial problems in their own.

Tutorials

There will be one tutorial per week. Tutorials start in Week 2.

Consultation

Please use e-mail to arrange an appointment with your tutor or the current lecturer.

Course Web Site

The MATH3161/MATH5165 course website will be available through UNSW Blackboard: http://telt.unsw.edu.au

UNSW Blackboard is accessed using your zID and zPass. Further information, problem sheets, lecture supplements, and other material will be provided via UNSW Blackboard.

Course Aims

The concept of optimization, finding the “best” way to do something, arises across all branches of mathematics and in application areas ranging from biology and engineering to business and finance. The purpose of this course is to provide an introduction to the theory of multi-variable optimization and optimal control, and to provide students with the skills to formulate, solve and analyze solutions to certain multi-variable optimization problems and infinite dimensional optimal control problems.
Relation to other mathematics courses
This course has a major focus on nonlinear continuous optimization problems, as distinct from linear programming problems and discrete optimization problems.

Teaching Strategies Underpinning the Course
New concepts and skills are first introduced and demonstrated in lectures, then students develop these skills by applying them to specific tasks in tutorials and assessments.

Rationale for learning and teaching strategies
We believe that effective learning is best supported by a climate of inquiry, in which students are actively engaged in the learning process. Hence, this course is structured with a strong emphasis on problem-solving tasks in lectures, tutorials and in assessment tasks, and students are expected to devote the majority of their class and study time to the solving of such tasks.
To ensure effective learning, students should participate in class as outlined below.

We believe that effective learning is achieved when students attend all classes, have prepared effectively for classes by reading through previous lecture notes, in the case of lectures, and, in the case of tutorials, by having made a serious attempt at doing for themselves the tutorial problems prior to the tutorials.

Furthermore, lectures should be viewed by the student as an opportunity to learn, rather than just copy down lecture notes.

Effective learning is achieved when students have a genuine interest in the subject and make a serious effort to master the basic material.

The art of logically setting out mathematics is best learned by watching an expert and paying particular attention to detail. This skill is best learned by regularly attending classes.

Expected Student Learning Outcomes
Students taking this course will develop an appreciation of the basic problems of optimization and skills to solve optimization problems. Computing skills are developed and practised in attempting assessment tasks.

By the end of the course students should be able to formulate, solve and analyze solutions to certain optimization problems. The ability to solve optimization problems via analytical, numerical and computational methods will be paramount.

Through regularly attending lectures, attempting assessment tasks, and applying themselves in tutorial exercises, students will develop competency in mathematical presentation, written and verbal skills.

Relation to graduate attributes: The above outcomes are related to the development of the Science Faculty Graduate Attributes, in particular: 1. Research, inquiry and analytical thinking abilities, 4. Communication, 6. Information literacy

Assessment
Assessment in this course will use problem-solving tasks of a similar form to those practised in lectures and tutorials, to encourage the development of the core skills underpinning this course and the development of analytical thinking.

There will be two (2) class tests, one (1) assignment and a final exam.
<table>
<thead>
<tr>
<th>Task</th>
<th>Date</th>
<th>Weighting</th>
<th>Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class Test 1</td>
<td>Tuesday, Week 6</td>
<td>15%</td>
<td>55 mins</td>
</tr>
<tr>
<td>Class Test 2</td>
<td>Tuesday, Week 11</td>
<td>15%</td>
<td>55 mins</td>
</tr>
<tr>
<td>Assignment</td>
<td>Tutorial, Week 12</td>
<td>5%</td>
<td></td>
</tr>
<tr>
<td>Final Exam</td>
<td>June</td>
<td>65%</td>
<td>2 hrs</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>100%</td>
<td></td>
</tr>
</tbody>
</table>

- **Class Tests** There will be two class tests counting 30% of the total assessment. Details of the class tests will be announced during lectures.
  - **Rationale** The Tests will give students feedback on their progress and mastery of the material.
  - There will be short answer questions in which correct answers are sought and there will be some longer questions requiring clear and logical presentation of correct solutions as well as some simple proofs and verbal explanations.
  - The Class Tests are held in place of the Tuesday lecture. Location for the class tests will be announced in lectures – it is your responsibility to get this information.

- **Final Exam:** The final exam, covering everything in the course, counts for 65% of the total assessment. Further details about the final examination will be available in class closer to the time.
  - **Duration:** Two hours.
  - **Rationale:** The final examination will assess student mastery of the material covered in lectures, tutorials, problems sheets, and any distributed material.

- **Starred Materials:** Problem sheets, class tests and the June exam may have starred questions indicating harder material. Grades of Pass and Credit can be gained by satisfactory performance on unstared questions. Grades of Distinction and High Distinction will require satisfactory performance on all questions.
  - Students in the graduate version (MATH5165) are expected to show satisfactory performance on starred questions.

- **Assignment** It is planned to have an assignment on solution methods of multi-variable optimization problems and optimal control problems. The assignment may involve modelling practical problems and writing a short report. The MATLAB software package may also be used for implementing numerical optimization methods to solve practical optimization problems. You are not required to know MATLAB before this course. On-line help in MATLAB is available by typing `help` or `help subject`. It is not assumed that you have done computing subjects.
  - Students in the graduate version (MATH5165) are expected to complete additional work in the assignment.
  - **Rationale:** Assignments will give an opportunity for students to try their hand at more difficult problems requiring more than one line of argument and also introduce them to aspects of the subject which are not explicitly covered in lectures. The assignment will also require a student to draw together several topics in the course.
  - Assignments must be YOUR OWN WORK, or severe penalties will be incurred.

You should consult the University web page on plagiarism

http://www.lc.unsw.edu.au/plagiarism
Additional Resources and Support

- **Calculators** You may bring your own UNSW approved Scientific Calculator to the class tests and final exam. Calculators will not be provided for you.

- **Text and Reference Books**
  There is NO textbook which covers all aspects of this course. General reference books are detailed in the last section.

- **Tutorial Exercises**
  Problem sheets for tutorials will be provided via UNSW Blackboard. These problems are for you to do to enhance mastery of the course.
  SOME of the problems will be done in tutorials, but you will learn a lot more if you try to do them before the tutorial.

- **Lecture Notes**
  A set of skeleton notes and summary sheets containing only definitions, theorems and proofs will be provided for SOME components of the course on UNSW Blackboard.

- **Library**
  The library has a mathematics subject guide on the web which is a good starting point for mathematical information. See
  http://subjectguides.library.unsw.edu.au/

Course Evaluation and Development

The School of Mathematics evaluates each course each time it is run. Feedback on the course is gathered, using among other means, UNSW’s Course and Teaching Evaluation and Improvement (CATEI) Process. Student feedback is taken seriously, and continual improvements are made to the course based in part on such feedback.

Administrative Matters

- **Special Consideration**
  Special consideration for class tests or the assignment will only be granted on medical or compassionate grounds and must be documented. If you miss a class test and you have strong medical reasons or other grounds, then contact the course authority, as soon as possible after the test with evidence. If you miss the June exam, then you must follow the School of Mathematics and Statistics standard procedures set out in the ‘Important Information for Students’ document. See
  http://www.maths.unsw.edu.au/currentstudents/student-services

- **Additional Assessment**
  The School of Mathematics has a strict policy on additional assessment. The School policy on special considerations for second and later years subjects will be strictly adhered to. Students must read and understand the School of Mathematics and Statistics Policies as contained in the ‘Information for Students’ document. See
  http://www.maths.unsw.edu.au/currentstudents/help-students-undergraduate
  http://www.maths.unsw.edu.au/currentstudents/help-students-postgraduate
• **Academic Misconduct**
  The University of New South Wales has rules relating to Academic Misconduct. See https://my.unsw.edu.au/student/academiclife/assessment/AcademicMisconduct.html

• **Rules for the conduct of examinations**
  The University of New South Wales has rules for the conduct of examinations. See http://www.maths.unsw.edu.au/currentstudents/assessment-policies

**Detailed Course Descriptions**

• **Overview:** Optimization is the study of problems in which we wish to optimize (either maximize or minimize) a function (usually of several variables) often subject to a collection of restrictions on these variables. The restrictions are known as constraints and the function to be optimized is the objective function. Optimization problems are widespread in the modelling of real world systems, and cover a very broad range of applications. Problems of engineering design (such as the design of electronic circuits subject to a tolerancing and tuning provision), information technology (such as the extraction of meaningful information from large databases and the classification of data), financial decision making and investment planning (such as the selection of optimal investment portfolios), and transportation management and so on arise in the form of a multi-variable optimization problem or an optimal control problem.

Optimization has its foundation in the development of calculus by Newton and Leibniz in the 17$^{th}$ century. The solution of large multi-variable optimization problems using computers started with the work of Dantzig in the late 1940s and 1950s on the simplex method for linear programming. Now, multi-variable optimization problems with hundreds of variables can be solved routinely.

• **Introduction:** What is an optimization problem? Areas of applications of optimization. Modelling of real life optimization problems.

• **Multi-variable optimization**
  – **Mathematical background:** Formulation of multi-variable optimization problems; Structure of optimization problems: objective functions and constraints. Mathematical background: multi-variable calculus and linear algebra; (strict) local and (strict) global minimizers and maximizers; convex sets, convex and concave functions; global extrema and uniqueness of solutions.
  – **Optimality conditions:** First and second order conditions for unconstrained problems; Lagrange multiplier conditions for equality constrained problems; Kuhn-Tucker conditions for inequality constrained problems.
  – **Numerical Methods for Unconstrained Problems:** Steepest descent method, Newton’s method, Conjugate gradient methods.
  – **Numerical Methods for Constrained Problems:** Penalty Methods.

• **Optimal Control**
  – **The Pontryagin maximum principle:** Autonomous control problems; unbounded controls.
Reference Books

The general references on optimization are listed below. The standard of the references is somewhat higher than is required in MATH3161/MATH5165.

Optimization References: General references on multi-variable optimization include [9, 3] and on optimal control include [7, 10]

Linear Algebra and Differential Equations: Solving multi-variable optimization problems requires techniques from linear algebra, whereas solving optimal control problems requires solution methods of differential equations. An elementary treatment of linear algebra can be found in Strang [12], while a reference for differential equations is Zill [13].


References