FACULTY OF SCIENCE

SCHOOL OF MATHEMATICS & STATISTICS

MATH3511
Transformations, Groups and Geometry

Semester 2, 2011
Course information
• 6 UOC
• Prerequisites: MATH1231 or MATH1241 or MATH1251

Course structure
There will be 3 lectures and one tutorial per week.

Course staff
A/Prof. Jie Du RC-4113, phone 9385-7087, email J.Du@unsw.edu.au
  o Office consultation: TBA, or by appointment.

Location and Times
• Lectures
  o Monday 11 – 12 OMB149
  o Wednesday 12 – 01 EE219
  o Friday 09 – 10 OMB G32
• Tutorials
  o Friday 10 – 11 OMB G32

Course description
A transformation on the plane is a bijection (or permutation) from the plane to itself. We will first
study several types of transformations such as translations, reflections, rotations etc. We will
also prove some theorems in classical Euclidean geometry and discover surprising properties of
triangles and circles. We will then look at symmetries, i.e. transformations of geometric figures
which preserve some property (such as distance or angles between lines) and projective
geometry, i.e. the study of perspective. Projective transformations can change a conic section of
one type to another, e.g. an ellipse to a hyperbola. This leads us then to the study of abstract
groups, where we will study the permutation group, subgroups, normal subgroups, direct
products, quotient groups and group homomorphisms.

Preparation:
You need to know some linear algebra, up to the level of MATH1241 or MATH1251 and some
basic naive set theory as you might pick up in those courses and, ideally a discrete maths
course. If you haven't done discrete maths, don't worry, the only "non-trivial" bits of set theory
you really need are the notions of products of sets and equivalence relations.

Expected Learning Outcomes
Students are expected to:
• understand the basic theory of transformation groups, and
• gain some knowledge of groups, especially symmetry groups.

Course Evaluation and Development
The School of Mathematics evaluates each course each time it is run. Feedback on the course
is gathered, using among other means, UNSW’s Course and Teaching Evaluation and
Improvement (CATEI) Process. Student feedback is taken seriously, and continual
improvements are made to the course based in part on such feedback.
**Assessment**
The grade for this course will be determined from 2 assignments (tentatively due in weeks 6 and 11 worth 10% each), 1 test (tentatively in week 8, worth 20%) and a final exam (worth 60%). Check the UNSW Blackboard website [http://lms-blackboard.telt.unsw.edu.au/](http://lms-blackboard.telt.unsw.edu.au/) for when the assignments are due.

**Syllabus**
The course will include material from the following. The course content is ultimately defined by the material covered in lectures.

**Part 1: Euclidean Geometry**

1: **Rigid Transformations of the plane**

- groups, isometries, geometric properties preserved by isometries, rotations, translations, reflections, glide reflections, direct & indirect isometries, relations between different types of isometries, group structure of the isometry group. Similarity, radial transformations, geometric properties preserved by similarities, group structure of the similarity group.

2: **Classical Theorems**

- Theorems of Ceva, Menelaus, etc. Theorems of centroid, orthocentre, etc.
- Nine-point circle theorem, Euler, Simson,

3: **Affine transformations**

- Affine transformations, geometric properties preserved by affine transformations, equiareal transformations, group structure of the group of affine transformations, fundamental theorem of plane geometry. Classifying geometric properties by transformation groups.

**Part 2: Projective geometry**

4: **Projections between planes in R^3**

- parallel and central projections between parallel or intersecting planes, writing affine transformations in terms of parallel projections, projecting circles to arbitrary conics, geometric properties preserved by central projections, cross-ratios of pencils and projections.

5: **The real projective plane**

- The extended Euclidean plane, points and lines at infinity, principle of duality, cross-ratios involving points at infinity, hemispheric model of the real projective plane, modelling the real projective plane by rays in space, homogeneous coordinates, lines and conics in the real projective plane.

6: **Projective transformations**

- central projections in terms of homogeneous coordinates, projective transformations, matrix representation, embedding the affine group in the group of projective transformations, group structure, geometric properties preserved by projective transformations, quadratic forms, polars and poles w.r.t. quadratic forms, transforming circle theorems to conic theorems.

**Part 3: Groups and symmetries**

7: **Basics from group theory**
Symmetry groups, Permutations, isomorphism, generators and relations, subgroups, homomorphisms, normal subgroups, cosets, factor groups, First isomorphism theorem, Lagrange's theorem.

8: Classification of some symmetry groups

Symmetries of a lattice, tesselation, fundamental region, theorem on classifying rotational symmetries for a lattice, point group, classifying point groups of a lattice.

References

The lectures, tutorials and problem sheets will cover all the material that you need to know, but nevertheless, you will probably find it handy to supplement your studies by looking at texts such as those below.

- Duzhin and Chebotarevsky, "Transformation Groups in Algebra, Geometry and Calculus", (relatively advanced level).

Library

- The library has a mathematics subject guide on the web which is a good starting point for mathematical information. They can be found at http://info.library.unsw.edu.au/ and http://info.library.unsw.edu.au/psl/guides/math/mathkey.html

Additional Assessment

- The School of Mathematics has a strict policy on additional assessment. It can be found at http://www.maths.unsw.edu.au/students/current/policies/studentpolicy.html

Plagiarism and academic integrity

- Plagiarism is the presentation of thoughts or work of another as one's own. Issues you must be aware of regarding plagiarism and the university's policies on academic integrity can be found at http://www.lc.unsw.edu.au/plagiarism and http://www.lc.unsw.edu.au/plagiarism/plagiarism_STUDENTBOOK.pdf

Academic Misconduct

- The University of New South Wales has rules relating to Academic Misconduct. They can be found at http://www.maths.unsw.edu.au/students/current/policies/studentpolicy.html.

Rules for the Conduct of Examinations

- The University of New South Wales has rules for the conduct of examinations. They can be found at http://www.maths.unsw.edu.au/students/current/policies/studentpolicy.html.

Occupational Health and Safety


Equity and Diversity

- Those students who have a disability that requires some adjustment in their teaching or learning environment are encouraged to discuss their study needs with the course convener prior to, or at the commencement of, their course, or with the Equity Officer (Disability) in the Equity and Diversity Unit (9385 4734 or www.equity.unsw.edu.au/disabil.html). Issues to be discussed may include access to materials, signers or note-takers, the provision of services and additional exam and assessment arrangements. Early notification is essential to enable
School of Mathematics Student Policies

School of Mathematics policy regarding tests, assignments additional assessment etc can be found at
http://www.maths.unsw.edu.au/students/current/policies/studentpolicy.html

You should at the very least make sure you are familiar with the "Important Information for Mathematics Students" that is linked there. The Plagiarism Policy is on the other side of this page.