Faculty of Science
School of Mathematics and Statistics

MATH3611 and MATH5705

HIGHER ANALYSIS

Semester 1, 2014
MATH3611 and MATH5705 – Course Outline

Information about the course

Course Authority/Lecturer:  Associate Professor Ian Doust
RC-5104, email i.doust@unsw.edu.au.

Consultation:  Times will be announced in week 2.

Credit & Prerequisites:  This course counts for 6 Units of Credit (6UOC).
- The prerequisites are 12 UOC of Level 2 Mathematics including MATH2111 or
  MATH2610 or MATH2011(CR) or MATH2510(CR) and an average mark of at least
  70, or permission from the lecturer. Those students who feel that they are borderline
  in meeting these prerequisites should discuss their position with the course authority
  and submit their first assignment early for review.
- Exclusions: MATH3570 Foundations of Calculus.
- This subject is a prerequisite for all students intending to do Honours in Pure
  Mathematics.

Lectures:  There will be three lectures per week:

<table>
<thead>
<tr>
<th>Thursday 3pm &amp; 4pm</th>
<th>in RC-4082</th>
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<tr>
<td>Friday 12</td>
<td>in OMB-G32</td>
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Tutorials:  There will be one tutorial per week, Friday 1pm in OMB-G32.
Note that it is expected that students will participate in tutorials more actively than
is sometimes the case in first and second year. It is vital that you have attempted
the tutorial problems before the tutorial, but the questions you raise should not
be restricted to those problems. If there is some concept that you don’t quite
understand, ASK!

Moodle:  Handouts should be available on Moodle.

Course aims

Real analysis is a central pillar of modern mathematics, and we will cover its foun-
dations. We start with the concepts of limits and continuity, which are at the core of
calculus, and we extend these concepts to quite general situations. The simplest case
(‘metric spaces’) is when there is some way of measuring the distance between two
points in the space. In the most important examples, a metric space occurs as a set
of functions, so we will look at ways in which one might say that a sequence of func-
tions converges. Taking these ideas one step further, we look at convergence which
does not come from a generalized distance function. These are the ideas of point set
topology. The course will also include topics such as countability, continuity, uni-
form convergence, compactness and connectedness, and measure and integration.
There is an introduction to central areas of functional analysis such as Banach and
Hilbert spaces. This is not a ‘computational’ course, but rather one in which you
will develop your ability to think and write abstractly, precisely and creatively.
Detailed course schedule

It is intended that the following topics will be covered. The actual number of lectures per topic may vary from that given here!

(1) Basics of set theory, (2 lectures)
   Cardinality, countability
   The Schroeder-Bernstein Theorem

(2) Metric spaces (7 lectures)
   Examples including sequence spaces and $C[0, 1]$.
   Topological properties of sets, open and closed sets, Convergence, completeness, closure and density, Normed spaces and inner product spaces. Contraction mapping theorem, Picard’s theorem from ODEs.

(3) Sequences and series of functions on metric spaces (3 lectures)
   Pointwise and uniform convergence of sequences of functions.
   Weierstrass M-test
   Differentiation and integration of limits and sums.

(4) Topological spaces (6 lectures)
   Open and closed sets, bases for topologies, separability and countability
   Convergence, Hausdorff topologies, generalised sequences
   Continuity and homeomorphisms
   Examples including weak topology, topology of local uniform convergence,

(5) Compactness (5 lectures)
   Review of Heine-Borel
   Continuous images of compact sets, compactness for metric spaces
   Uniform continuity, Weierstrass’ approximation theorem
   Tychonoff theorem,
   Arzela’s theorem.

(6) Connectedness (1 lecture)
   Path-connectedness, continuous images of connected sets

(7) The basics of measure and integration (3 lectures)
   Basic measure theory, Lebesgue measure
   Integration, dominated convergence theorem.

(8) Banach spaces and Hilbert spaces (3 lectures)
   Convergence in $L^p$ spaces
   Review of examples
   Generalising weak convergence: dual spaces; the duals of standard spaces
(9) Linear operators on Banach spaces (4 lectures)
   Examples: infinite matrices, integral operators, translations, multiplications
   Continuity and boundedness
   Projections

(10) Reproducing Kernel Hilbert Spaces (2 lectures)
   When does $L^2$ convergence imply pointwise convergence?
   Examples

Assessment

Assessment in this course will consist of 4 rather short assignments (worth 5% each), a longer assignment (worth 20%), and a 2 hour final examination (60%).

Students enrolled in MATH5705 will be expected to perform at a higher level than those in MATH3611 in order to achieve a Distinction or High Distinction. Such students may need to attempt additional problems on the main assignment and the final exam.

Assignments

Due dates for these are:

<table>
<thead>
<tr>
<th>Task</th>
<th>Date Due</th>
<th>Weighting</th>
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<tbody>
<tr>
<td>Ass 1</td>
<td>Midday Fri Week 2</td>
<td>5%</td>
</tr>
<tr>
<td>Ass 2</td>
<td>Midday Fri Week 4</td>
<td>5%</td>
</tr>
<tr>
<td>Ass 3</td>
<td>Midday Fri Week 6</td>
<td>5%</td>
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<tr>
<td>Ass 4</td>
<td>Midday Fri Week 8</td>
<td>5%</td>
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<tr>
<td>Main Ass</td>
<td>Midday Fri Week 10</td>
<td>20%</td>
</tr>
<tr>
<td>Exam</td>
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<td>60%</td>
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Assignments will give an opportunity for students to try their hand at more difficult problems requiring more than one line of argument and also introduce them to aspects of the subject which are not explicitly covered in lectures. They will enable students to consolidate their understanding of topics as we go which will ensure that later material does not become inaccessible.

Assignments must be written up using a high standard of presentation. Attention must be paid to spelling and grammar as well as explaining the argument. All assignments must be ‘typed’, that is, prepared using (preferably) \LaTeX, Word or some other suitable package.

[Students doing MATH5705 are expected to uphold a particularly high level of presentation and explanation.]

Late assignments will attract a significant late penalty, and will not be accepted after solutions have been posted.
Examination
The final examination will assess student mastery of the material covered in the lectures. The exam will include questions requiring knowledge of the definitions, examples and theorems in the course, as well as problems to be solved using such knowledge.

**Duration:** Two hours.  **Weighting:** 60% of your final mark.

Further details about the final examination will be available in class closer to the time.

Additional resources and support

Problem Sheets
A set of problem sheets will be given out. These problems are for you to do to enhance mastery of the course. SOME of the problems will be done in tutorials, but you will learn a lot more if you try to do them before the tutorial. Some of the problems will be chosen as assignment questions for the fortnightly assignments.

Lecture notes
Some notes may be provided on the course website from time to time.

Textbooks
The set textbook for this course is:

- A.N. Kolmogorov and S.V. Fomin: Introductory Real Analysis (Dover, 1970; Call number: P517.5/125).

This covers most of the material for this course and also for some of the later courses. It has the benefit of being quite cheap, but the disadvantage of sometimes using slightly old-fashioned terminology. In case of a conflict, the ‘official’ definition is the one given in lectures!

**Other recommended books:**

- W. Rudin: Principles of Mathematical Analysis
- G. F. Simmons: Introduction to Topology and Modern Analysis
Course Evaluation and Development

The School of Mathematics and Statistics evaluates each course each time it is run. We carefully consider the student responses and their implications for course development. It is common practice to discuss informally with students how the course and their mastery of it are progressing.

On the basis of past CATEIs we have introduced MATH2711 as a stepping stone to this course, and will be concentrating on meeting the needs of a wider range of students.

Student Learning Outcomes

Students taking this course will develop an appreciation of the basic concepts of Real and Functional Analysis, including the study of metric spaces, topological function spaces and operator theory. These methods will be useful for further study in a range of other fields, e.g. Quantum Theory, Stochastic calculus and Harmonic analysis.

Relation to graduate attributes: The above outcomes are related to the development of the Science Faculty Graduate Attributes, in particular: 1. Research, inquiry and analytical thinking abilities, 4. Communication, 6. Information literacy

Teaching strategies underpinning the course

New ideas and skills are introduced and demonstrated in lectures, then students develop these skills by applying them to specific tasks in problem sheets and assessments.

Rationale for learning and teaching strategies: We believe that effective learning is best supported by a climate of inquiry, in which students are actively engaged in the learning process. Hence this course is structured with a strong emphasis on problem-solving tasks in assessment tasks, and students are expected to devote the majority of their class and study time to the solving of such tasks.

Assessment criteria: The main criteria for marking all assessment tasks will be clear and logical presentation of correct solutions.

Knowledge and abilities assessed: All assessment tasks will assess the learning outcomes outlined above.

Administrative matters

Details of the general rules regarding attendance, release of marks, special consideration etc are available via the School of Mathematics and Statistics Web page. You should particularly check the Student Services page at

http://www.maths.unsw.edu.au/currentstudents/student-services
Students should particularly note that special additional assessment policies that apply to students doing MATH2xxx and MATH3xxx courses whose final mark is in the range 40–49.

**Equity and diversity issues**

Students with any special needs and requests should either see the lecturer as soon as possible, or ensure that SEADU sends appropriate documentation.

Similarly, any student whose study is being adversely affected by external issues should see the lecturer and/or get an appropriate counsellor/doctor/… to contact the lecturer.

**Plagiarism and academic honesty**

Assignments must be YOUR OWN WORK, or severe penalties will be incurred. You should consult the University web page on plagiarism. Plagiarism is the presentation of the thoughts or work of another as one’s own.

Issues you must be aware of regarding plagiarism and the university’s policies on academic honesty and plagiarism can be found at

http://student.unsw.edu.au/plagiarism, and


You should be also be aware that plagiarism rules vary significantly from discipline to discipline. Some issues that are mathematics specific are discussed below.

The plagiarism rules do not preclude you from consulting references and discussing questions with other people. Indeed you can often learn a great deal from such discussions. It just means that you need to be clear and honest about where the ideas came from. This can usually be covered by a sentence such as ‘This proof is based on the proof of Theorem 3.2.1 in …’ or ‘The clever idea to consider random subsets of Γ was provided by Fred’.

- **You will not lose marks because you found the answer in a book (great research!) or were helped by a friend.**

- **You will lose marks if you don’t say where you found a solution, or who helped you, or if you copy something without understanding what you have written.**

You don’t need to reference definitions, theorems etc that you would expect everyone in the course to know (eg the Cauchy Integral formula, or the definition of an analytic function, or something from the lectures). On the other hand, if your proof depends on a slightly more unusual theorem (eg Jensen’s formula), then you should (a) check that that theorem doesn’t depend on the thing that you have been asked to prove, and (b) give a reference to where you found the result. Your solutions should be written so that a fellow student could read and follow what you have done — keep this in mind as you are writing up your work.
Obviously straight copying of someone else’s assignment is both unethical and easily detected. You should never provide your written solutions to another student; inevitably their work will look just like yours and you’ll both be penalized. If you work on a problem with a friend, make sure that you each write it up quite separately (and mention that you discussed the problem with your classmate).

Similarly, you should never copy word-for-word something that find in a text. In mathematics there is a limit to how much one can rearrange a proof to make it look like yours so we allow greater degrees of similarity than would be acceptable in many other disciplines. However it is almost never the case that you find a proof that has the same sort of notation that we are using, exactly the same definitions and theorems etc. You always need to adapt these things to the context of the course that you are doing.

It is very easy to tell when someone has copied something and not understood it. Serious cases of plagiarism will attract severe penalties.