Faculty of Science
School of Mathematics and Statistics

MATH3701

HIGHER TOPOLOGY AND DIFFERENTIAL GEOMETRY

Semester 2 2013

CRICOS Provider No: 00098G

Information about the course

Course Authority: Assoc.Prof. Norman Wildberger, Room RC-4108, phone 9385 7098, email `n.wildberger@unsw.edu.au`.

Consultations times will be announced in lectures.

Course Information: This is a 6 unit of credit course with one lecture stream. Prerequisites: 12 units of credit of Level II Mathematics courses with an average mark of 70 or higher, including MATH2111 or MATH2011 (Credit) or MATH2510 (Credit) and MATH2601 or MATH2501 (Credit), or permission from Head of Department. Exclusions: MATH3690, MATH3700, MATH3531, MATH3760.

Lectures: Monday 1pm–2pm (OMB-G32); Wednesday 2pm–3pm (ME-405), Thursday 4pm–6pm (OMB-G32), weeks 1:9 and 11:13.

Tutorials arrangements: Roughly every fourth meeting will be run as a tutorial. I expect students to participate in tutorials more actively than is sometimes the case in first and second year.

Course aims

The principal aim of this subject is to introduce students to the topology and differential geometry of curves and surfaces, and to study some of the many applications.

Relation to other mathematics courses

This course builds on the material in courses in Several Variable Calculus (Math2011/2111) and Linear Algebra (Math2501/2601). It covers material relevent to many branches of mathematics, but is especially relevent to students with an interest in theoretical physics.

Student Learning Outcomes

By the end of the course, you should have a good understanding of basic topological properties, constructions and reasoning in three dimensional space, classical curves and surfaces, and understand the meaning of curvature for curves and surfaces, and appreciate the connections between topology and differential geometry for surfaces. You should also know something of the mathematics behind general relativity and other physical theories that arise from concepts of differential geometry. You will gain an appreciation for the importance of quadrics to approximate surfaces at a point, and you will be able to make explicit computations for a wide variety of examples, computing Frenet frames for curves, and first and second fundamental forms for many surfaces. Algebraic surfaces and surfaces of revolution will provide a good source of examples. You will understand the idea of a developable surface and its applications.

Relation to graduate attributes

The above outcomes are related to the development of the Science Faculty Graduate Attributes, in particular: 1.**Research, inquiry and analytical thinking abilities**, 4. **Communication**, 6. **Information literacy**

Assessment

Assessment in this course will consist of three assignments and a final exam. The assignments are of equal weight and worth in total 60%. The final exam will be worth 40%.

The assignments will be given out by weeks 3, 6 and 9, and due in weeks 5, 9 and 12 (precise dates on the assignments).

Assignments

Submission: You must submit assignments via a hard copy (paper) only. You will have to sign the University's anti-plagiarism declaration for each assignment, declaring that the assignment is your own work. See page 5 of this document for further information on what consitutes plagiarism.

Rationale: The main rationale for the assignments is to give you practice and feedback in the theory and applications of the ideas from the lectures and to improve your communication skills, so solutions should be written out carefully and neatly.

Marking criteria: Although the primary criteria will be mathematical correctness, some marks will be explicitly awarded for presentation, and marks will be deducted for sloppiness in presentation, grammar, spelling and so forth.

If you find a solution in a book, that is fine, but be sure to give a complete reference for where you found it. You should also take care that you are using the same sort of terminology and notation as we are using in this course.

You may discuss the problems with the other students, but **never** show anyone a copy of your solution. Marks will be deducted if I get two solutions which are closer than appears reasonable. (No attempt will be made to determine who copied from whom!)

You may use Maple (or matlab) for long or tedious calculations, as long as a relevent (and suitably annotated) printout is included.

Late assignments: Unexcused late assignments will not be accepted. If you are ill, or have some other problem which will result in late assignments, then you must let me know the problem **before** class on the day the work is due.

Knowledge and abilities assessed: All assessment tasks will assess the learning outcomes outlined above, specifically, the ability to apply the definitions and techniques from lectures to solve problems, prove theorems and present clear proofs.

Assessment criteria: The main criteria for marking all assessment tasks will be clear and logical presentation of correct solutions. Concise solutions are appreciated, but they must also be complete.

Examination

Duration: Three hours.

Rationale: The final examination will assess student mastery of all the material covered in the lectures, except for any material specifically identified as non-examinable (there may be no such material).

Further details about the final examination will be available in lectures.

Final Mark

Your (preliminary) final mark out of 100 will be a scaled version of the raw mark formed by combining the assignments and exam.

Additional resources and support

Tutorial Exercises

The problem sheets will be available on Moodle in the form of PDF documents.

Lecture notes

Students are expected to attend class and take their own lecture notes. Taking careful and accurate notes is an important skill. Please use a separate A4 notebook.

Textbooks

There are no recommended or required textbooks for this course — my approach is not the standard classical one. Parts of a number of texts available in the library may be useful. I will mention useful references in the lectures.

Computing

All students in mathematics courses are given a computing account in the School of Mathematics and Statistics and may use the PCs in the School of Mathematics Computing Laboratories in M020 and G012 in the centre wing, Red Centre. Log in to these computers using your zID and zPass. For further information on these labs, see the information on the School's website.

`http://www.maths.unsw.edu.au/currentstudents/computing-information`

There is **no** formal computing component in the MATH3701 course. There are no references to Maple (or matlab) in the course problem booklet and no assessment will require Maple or matlab. However, I expect you to explore the use of Maple or matlab to help develop your three dimensional visualisation skills.

Administrative matters

Students must read and understand the School of Mathematics and Statistics Policies contained in the 'Important Information for Students' document. This can be found on the web at

`http://www.maths.unsw.edu.au/currentstudents/help-students-undergraduate`

Additional Assessment

Please read the important information regarding additional assessment on the School's website at:

`http://www.maths.unsw.edu.au/currentstudents/additional-assessment`

School Rules and Regulations

Fuller details of the general rules regarding attendance, release of marks, special consideration etc are available via the School of Mathematics and Statistics Web page at:

`http://www.maths.unsw.edu.au/currentstudents/help-students-undergraduate`

Plagiarism and academic honesty

Plagiarism is the presentation of the thoughts or work of another as one's own. Issues you must be aware of regarding plagiarism and the university's policies on academic honesty and plagiarism can be found at

`http://www.lc.unsw.edu.au/plagiarism`

and

`http://www.lc.unsw.edu.au/plagiarism/plagiarism_STUDENTBOOK.pdf`

Rough list of topics (likely to be modified)

1. Introduction and overview
2. Point Set Topology: topological spaces; continuous functions, homomorphism; homotopy; topological properties; quotient topology
3. Differential Geometry of Curves: classical curves and differential approximations; curvature and torsion; Serret-Frenet equations.
4. Topological Properties of Curves; winding numbers and turning numbers; vector fields and index; The Poincaré Index Theorem.
5. Surfaces: parametrized surfaces, quadric approximations, fundamental forms, shape operators

6. Curvature: Gauss' *theorema egregium*; minimal surfaces, surfaces of revolution, ruled surfaces, geodesics; The Gauss-Bonnet theorem;

7. Topology and Surfaces: polygonal curvature; Euler characteristic; Poincaré Index Theorem on surfaces; topological surfaces.

8. Differentiable Manifolds: abstract surfaces; hyperbolic plane; Riemannian geometry; general relativity.

Teaching strategies underpinning the course

I first introduce and demonstrate new ideas and skills in lectures. You are then expected to develop these skills by applying them to specific tasks in tutorials and assessments. In this way, and also by your persuing some lines of inquiry suggested by the course material, I expect to further your research, inquiry and analytical thinking abilities and also your capability and motivation for intellectual development

Rationale for learning and teaching strategies

I believe that effective learning is best achieved by a climate of enquiry, in which students are actively engaged in the learning process. I expect students in this course to devote the majority of their class and study time to applying the techniques learnt in lectures to specific cases in tutorials and in assessment tasks. This means I expect you to do the tutorial problems and attend the lectures and tutorials.

You should plan to spend about 4-6 hours per week on reading and homework, and plan a **regular study program** for yourself. If you do not study regularly you will rapidly become lost, as much of the later part of the course builds substantially on the earlier. As a higher third year course, it must be treated seriously.

You will learn most effectivly when you have a genuine interest in the subject and make a serious effort to master the basic material.

The art of logically setting out mathematics is best learned by watching an expert and paying particular attention to detail. This skill is also best learned by regularly attending classes and taking good notes.

Course Evaluation and Development

The School of Mathematics and Statistics evaluates each course every time it is run. We carefully consider the student responses and their implications for course development, and look forward to your comments and suggestions this time around!