FACULTY OF SCIENCE

SCHOOL OF MATHEMATICS AND STATISTICS

MATH3711 Higher Algebra
and
MATH5706 Modern Algebra

SEMESTER 1, 2017

Course information

- 6 UOC
- Prerequisites (MATH3711 only): 12 UOC of Level 2 Mathematics with an average mark of at least 70, including MATH2601 or MATH2501 (CR), or permission from the Head of Department

Course staff

Dr David Harvey, RC-6108, phone 9385-7088, email d.harvey@unsw.edu.au

Consultation with lecturer is by appointment.

Location and Times

- Lectures
  - Thursday 10 am – 12 pm RC-2060
  - Friday 10 am – 11 am RC-1043
- Tutorials
  - Friday 11 am – 12 pm RC-1043

Course description

This course introduces the basic language and ideas of modern algebra, the two main concepts being that of a group and that of a ring. Groups are the vehicle through which mathematicians study symmetry. The data used to define them involves one algebraic operation, group multiplication. Rings on the other hand have two algebraic operations, addition and multiplication, much as you find with ordinary numbers. Historically, solving polynomial equations was a major driving force for the development of group theory, while number theory was the major driving force for ring theory.

Preparation

You need to know some linear algebra, up to the level of higher first year mathematics courses (MATH1241 or MATH1251), and some basic naive set theory as you might pick up in those courses, and ideally a discrete mathematics course. If you haven't done discrete mathematics, don't worry, the only “non-trivial” bits of set theory you really need are the notions of products of sets and equivalence relations. You can easily find them in a reference book listed below, e.g., Chapter 0 of Jacobson’s book. Lastly, and most importantly, you need a certain amount of mathematical maturity, which the handbook attempts to define as 12 units of credit in level 2 maths courses with a distinction average. Certainly, if your average is 70 or lower, you should come and see me. Most of you will have done the 2nd year linear algebra course MATH2601 and already seen a little group theory. We will go over all this material, but only lightly. More precisely, the theory will be covered, but there will be few examples so you should check out the relevant tutorial questions etc carefully if this material is unfamiliar to you.

Expected Learning Outcomes

Students are expected to:

- understand the basic theory of groups, rings and fields; and
- increase their abilities in writing proofs in good mathematical style.

This course is as hard as it is interesting. It is very demanding conceptually and indeed, the lectures will introduce lots of new ideas and modes of thinking that will take some time to get used to. Most likely you will have to expand and develop your learning patterns to cope. See some tips in a separate handout to help you out. Your first and foremost goal will be to understand the material. Make sure you try to fill in gaps in your understanding after each lecture. I expect this will take up a sizeable chunk of your study for this course. The tutorials are designed to reinforce your understanding of the material and also to develop your problem solving skills.

Course Evaluation and Development

The School of Mathematics evaluates each course each time it is run. Feedback on the course is gathered, using among other means, UNSW's Course and Teaching Evaluation and Improvement (CATEI) Process. Student feedback is taken seriously, and continual improvements are made to the course based in part on such feedback.

Assessment

The grade for this course will be determined from three written assignments (tentatively due in week 3, week 7 and week 12, worth 15%, 15% and 20% respectively), and a final exam (worth 50%). The assessments for MATH3711 and MATH5706 are the same, but students enrolled in MATH5706 will be expected to perform at a higher level than those in MATH3711 in order to achieve a Distinction or High Distinction. Check the UNSW moodle website

https://moodle.telt.unsw.edu.au/

for when the assignments are due (as well as for hints and typos!).

I encourage you to submit your assignment solutions in TeX, but handwritten assignments are also acceptable. All assignments must be submitted as hard copies (please do not email assignments). You are welcome to discuss the assignment problems with me or with other students, but you must write up the solutions yourself. Any collaboration with other students must be noted on your submission.

A major aim of the assignments is to develop your mathematical writing skills. A solution that is mathematically completely correct but poorly organised or communicated will not receive full marks. *If it takes me a long time to read and understand your solution, it is probably not written very clearly.* Therefore I encourage you to make my job as easy as possible, by making your solution as easy to read as possible.

Syllabus

The course will include material from the following. The course content is ultimately defined by the material covered in lectures.

1: Examples of groups (5 lectures)

- How do mathematicians study symmetry
- Matrix groups, permutation groups, Abelian groups, dihedral groups
- Subgroups, generators and relations

2: Basic concepts and constructions (6 lectures)

- Cosets and Lagrange's theorem, isomorphism and homomorphism
- Normal subgroups and quotient groups
- Isomorphism theorems
- Direct products and sums

3: Some classification results (4 lectures)

- Classification of cyclic groups, finite abelian groups
- Classification of some plane and space isometry groups

4: Group actions and counting with groups (3 lectures)

- Groups actions, orbits and stabilisers
- Classification of orbits, counting formula

5: Introduction to ring theory (7 lectures)

- Rings and subrings, examples
- Ideals and quotient rings, homomorphism and isomorphism theorems
- Commutative rings and fields

6: Factorisation in commutative domains (5 lectures)

- Primes and irreducibility, Gaussian integers
- Unique factorisation domains, principal ideal domains, Euclidean domains
- Gauss' lemma

7: Fields (4 lectures)

- Fields and field extensions
- Algebraic and transcendental extensions, algebraic closure
- Finite fields
- Straightedge and compass constructions

References

Lecture notes will generally be provided one week in advance. The lecture notes and other course materials have been developed over several years by various staff members (including Daniel Chan, Jie Du, and David Harvey).

The lectures, tutorials and problem sets will cover all the material that you need to know, but nevertheless, you will probably find it handy to supplement your studies by looking at texts such as those below. Some of them only cover the group theory portion of the course. There are lots of texts designed for a first course in algebra. They vary a lot so you should scout around for what's suitable for you.

- Artin, *Algebra*
- Armstrong, *Groups and Symmetry*
- Rotman, *A first course in abstract algebra*
- Herstein, *Topics in Algebra*
- Jacobson, *Basic Algebra I*
- Stillwell, *Elements of Algebra*
- Lang, *Algebra*
- Lederman, Weir (Jeffrey), *An Introduction to Group Theory*

Library

The library has a mathematics subject guide on the web which is a good starting point for mathematical information. They can be found at http://info.library.unsw.edu.au/ and http://info.library.unsw.edu.au/psl/guides/math/mathkey.html

Additional Assessment

The School of Mathematics has a strict policy on additional assessment. It can be found at http://www.maths.unsw.edu.au/students/current/policies/studentpolicy.html

Plagiarism and academic integrity

Plagiarism is the presentation of thoughts or work of another as one's own. Issues you must be aware of regarding plagiarism and the university's policies on academic integrity can be found at

http://www.lc.unsw.edu.au/plagiarism and http://www.lc.unsw.edu.au/plagiarism/plagiarism_STUDENTBOOK.pdf

Academic Misconduct
The University of New South Wales has rules relating to Academic Misconduct. They can be found at http://www.maths.unsw.edu.au/students/current/policies/studentpolicy.html.

Rules for the Conduct of Examinations
The University of New South Wales has rules for the conduct of examinations. They can be found at http://www.maths.unsw.edu.au/students/current/policies/studentpolicy.html.

Occupational Health and Safety
Occupational Health and Safety policies and expectations: www.riskman.unsw.edu.au/ohs/ohs.shtml

Equity and Diversity
Those students who have a disability that requires some adjustment in their teaching or learning environment are encouraged to discuss their study needs with the course convener prior to, or at the commencement of, their course, or with the Equity Officer (Disability) in the Equity and Diversity Unit (9385 4734 or www.equity.unsw.edu.au/disabil.html). Issues to be discussed may include access to materials, signers or note-takers, the provision of services and additional exam and assessment arrangements. Early notification is essential.

School of Mathematics Student Policies

School of Mathematics policy regarding tests, assignments additional assessment etc can be found at

http://www.maths.unsw.edu.au/students/current/policies/studentpolicy.html

You should at the very least make sure you are familiar with the “Important Information for Mathematics Students” that is linked there. The Plagiarism Policy is on the other side of this page.