Never Stand Still | Science | School of Mathematics and Statistics

Course Outline

MATH5535 Special Topics – Lie Groups

School of Mathematics and Statistics

Faculty of Science

Semester 2, 2016

SCHOOL OF MATHEMATICS AND STATISTICS
UNIVERSITY OF NEW SOUTH WALES

MATH5535 Special Topics — Lie Groups Semester 2 2016

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This is a 6 UOC level V course, suitable for students who have taken several years of a variety of University mathematics courses. This includes Honours students, coursework Masters students and undergraduates who have completed 24 units of level III mathematics. Other (motivated and capable) students may enrol with the permission of the Lecturer-in-Charge.

There is scheduled to have three lectures per week in weeks 1 to 12. These will be held in RC-1043 on Tuesday 4pm–6pm and Wednesday 12noon–1pm. However, it is likely that some additional lectures will be scheduled during weeks 1 to 11 so that class can be finished by the end of week 11 as the lecturer is expected to be away for weeks 12 and 13.

About this course and course aims

While studying differential equations, Lie was lead to consider the action of transformations preserving a differential equation, that is, its symmetries. Since symmetries can be composed and reversed, they form a group. Lie considered group actions in which both the symmetries and the spaces on which they acted were parameterised by open sets in $\mathbb{R}^n$ or $\mathbb{C}^n$ and so inherited, at least locally, the standard topology of $\mathbb{R}^n$ or $\mathbb{C}^n$. Moreover, the group multiplication and inverse operations were smooth functions. That is, Lies groups had the structure of both a group and a smooth manifold and these structures were compatible. Locally, the structure of a Lie group is captured by the the tangent space at the identity element which leads to the study of Lie algebras. The study of Lie groups and Lie algebras was developed throughout the latter part of the 19^{th} century and early 20^{th} century by Lie, Killing, Engel and others leading to Cartans classification of simple Lie algebras. In this course we will cover the basic definitions required to understand Lie groups and Lie algebras and see how the compatibility of the group and topological (and smooth) structures induces a rich mathematical structure. The main aims of this course are to introduce basic results in Lie groups along with examples and applications and ultimately to describe the classification of simple Lie groups via the classification of simple Lie algebras in terms of Dynkin diagrams.

Student learning outcomes

Students taking this course will:

- develop an understanding of the definition, properties and applications of Lie groups and Lie algebra,
- develop their ability to perform calculations with Lie groups and Lie algebra,
- understand the classification of semi-simple Lie algebras.

These outcomes particularly relate to Faculty of Science Graduate Attribute 1: *Research, inquiry and analytical thinking abilities* and UNSW Graduate Attribute 3: *the capacity for analytical and critical thinking and for creative problem solving.*

Teaching strategies used

New concepts and techniques are first introduced and demonstrated in lectures, then students master these concepts and techniques by applying them to the problem sets and to assessment tasks. In lectures, students will be expected to think, as well as listen, and will have the opportunity to test their understanding by answering questions posed by the lecturer.

Rationale: We believe that effective learning is best supported when students are actively engaged with the new mathematical concepts and techniques, for example by thinking about the new material, asking questions during lectures and making a serious attempt to solve the problems.

Assessment

The plan is to have 2 assignments worth 20% each, due in weeks 6 and 10, and a final exam worth 60%. This plan will be discussed at the first lecture.

Assessment criteria: In the assignments and the exam, marks will be awarded for correct working, logical setting out, appropriate explanations, clear notation and presentation, as well as for the final answer. The aim of this is to develop students' ability to present their mathematics in a professional way.

Assessment rationale: Assessment in this course will evaluate the students' understanding of the combinatorial concepts and methods presented in lectures (Science Graduate Attribute 1) and their mastery of problem-solving techniques developed in lectures, as well as creativity and critical thinking (UNSW Graduate Attribute 3). The assignments will also provide feedback on students' progress and may introduce new concepts not covered in lectures.

Assignments

Assignments must be typed using a word processor or, preferably, prepared using the mathematical typesetting language LaTeX. Assignments must be submitted via Moodle using the link provided. Unless an extention is approved in advance, there will be a 10% penalty per day late up to a maximum of one week.

Students may discuss solutions to assignment questions with other students currently taking the course, *provided* that they write up their solution independently (and not simply copy from each other) and acknowledge help that they have received from fellow students or from books (giving a reference). Note that Turnitin will be used to check the assignments against its bank of previously submitted assignments and internet sources for similarities.

Final exam

The final exam will cover material from the whole course. Further information about the exam will be given out in lectures towards the end of the course.

Syllabus

The topics will be covered in the following order, but note that the indicated weeks (in brackets) are (very!) approximate.

1. **Review of smooth manifolds (2 weeks)**
   Basic definitions, smooth structure, tangent and cotangent bundles, derived maps, product manifolds, sections, tensors
2. **Lie Groups (2 weeks)**
   Definitions, examples, left/right translation, adjoint map, Lie subgroups, homomorphisms, connected component of the identity, semi-direct products, covering groups.
3. **Lie Algebras (2 weeks)**
   Definitions, examples, adjoint action, Lie algebra of a Lie group, left invariant vector fields, Lie algebra of matrix Lie groups, Ado's theorem, one parameter subgroups, exponential map, universal enveloping algebra, Poincare-Birkhoff-Witt theorem, representations.
4. **Group actions (2 weeks)**
   Group action on a manifold, equivariant rank theorem, equivariant map, orbit map, isotropy subgroups, transitive group action, quotient manifold, homogeneous spaces.
5. **Classification of Lie algebras (4 weeks)**
   Ideals and normal subgroups, simple Lie algebra, lower central series, derived series, nilpotent, solvable, semisimple, radical of a Lie algebra, Engel Theorem, Lie's Theorem, reducibility, maximal torus, root spaces, Killing form, Cartan criterion, real forms, compact Lie algebra, Cartan matrix, Dynkin diagrams, finite-dimensional representations of $\mathfrak{sl}(2)$.

Additional resources

Textbooks: The content of the course will be defined by the lectures. There is no set text for this course, however there are many reference books. Some which may be useful (for certain parts of the course) are:

- J.F. Adams, *Lectures on Lie Groups*, University of Chicago Press.
- S. Helgason, *Differential Geometry, Lie Groups, and Symmetric Spaces*, American Mathematical Society.
- F. Warner, *Foundations of Differentiable Manifolds and Lie Groups*, Springer.
- A. Sagle and R. Wadle, *Introduction to Lie Groups and Lie Algebras*, Academic Press.
- W. Fulton and J. Harris, *Representation Theory: A First Course*, Springer.
- J. M. Lee, *Introduction to Smooth Manifolds*, Springer.

There are many sets of notes available on the internet. Some suggested links will be posted on Moodle.

Course evaluation and development

The School of Mathematics and Statistics evaluates each course each time it is run. Feedback on the course is gathered, using among other means, UNSW's Course and Teaching Evaluation and Improvement (CATEI) Process. Student feedback is taken seriously and continual improvements are made to the course based in part on such feedback.

Administrative matters

- The School of Mathematics and Statistics has policies regarding attendance, additional assessment, special consideration in the event of illness and misadventure, and so on. *We assume that you are familiar with these policies, so please familiarise yourself with them!* These policies can be found by following links from these pages:

  `http://www.maths.unsw.edu.au/currentstudents/student-services.html`
  `https://student.unsw.edu.au/policy`

  Note that the Pass Conceded (PC) grade is no longer given at UNSW. As a result, the School has a new policy that a concessional additional assessment exam will be offered to students whose final grade is in the range 45 to 49. Students taking this concessional additional assessment exam will will have their final grade capped at 50. For more information, see the policies on the School's website.

- You should also know what plagiarism is and be aware of UNSW's plagiarism policy and student student misconduct. See:

  `https://student.unsw.edu.au/plagiarism`
  `https://student.unsw.edu.au/conduct`

- UNSW has advice for students regarding health and safety:

  `https://student.unsw.edu.au/wellbeing`

- If you are managing the demands of university as well as a health condition, learning disability or have personal circumstances that are having an impact on your studies, disability services may be able to provide you with assistance.

  `https://student.unsw.edu.au/disability`