Faculty of Science
School of Mathematics and Statistics

MATH5605

FUNCTIONAL ANALYSIS

SEMESTER 1, 2013
MATH3521 – Course Outline

Information about the course

Course Authority:  Professor Michael Cowling

Lecturer:  Professor Michael Cowling RC-5113, email m.cowling@unsw.edu.au.

Consultation: Please contact the lecturer before or after lectures, or use email to arrange an appointment.

Credit, Prerequisites, Exclusions:

This course counts for 6 Units of Credit (6UOC).

There are no formal prerequisites for this course, but you will need to have seen quite a bit of linear algebra, such as in MATH2601 and some analysis, such as in MATH3611, and to have achieved good marks in these courses.

If you are not an Honours student, you will need permission to take the course from the Director of Undergraduate Studies or the Head of Department, and will be enrolled manually—you cannot enrol yourself. As a first step, contact the course authority, Michael Cowling.

Exclusions: none.

There is no ordinary version of this subject.

Lectures: There will be four lecturers per week for the first nine weeks: timetable to be arranged on Monday Week 1 at 9.00 am in RC-4084–4085.

Tutorials: At least one of the lectures will function as a tutorial each week. This will depend on the topics under consideration.

Blackboard: Further information and other material will be provided via the University’s on-line learning system.

Course aims

This course introduces functional analysis. This area combines ideas from linear algebra and analysis in order to handle infinite-dimensional vector spaces and linear mappings thereof.

Relation to other mathematics courses

Pure mathematics may be divided into the broad categories of analysis (calculus), algebra, geometry and logic. This subject combines ideas from analysis and linear algebra, and underpins many mathematical developments of the last hundred years,
including many aspects of partial differential equations, mathematical physics and probability, and the area of mathematics often called Modern Analysis.

This course is useful for those working in many areas of mathematics, including Mathematical Physics, Partial Differential Equations, Mathematical and Numerical Analysis, and pure or applied Probability.

Student Learning Outcomes

At the end of this course, you will be able to appreciate how ideas from different areas of mathematics combine to produce new tools that are more powerful than would otherwise be possible, and you will understand how functional analysis underpins modern analysis. You will have developed:

- your mathematical intuition and your problem-solving capabilities;
- your understanding of which tool is appropriate to tackle which problem;
- your ability to find information through tools like the world-wide web to solve problems;
- your ability to use computers to illustrate your arguments;
- your competency in mathematical presentation, and your written and verbal skills.

Relation to graduate attributes

The above outcomes are related to the development of the Science Faculty Graduate Attributes, in particular: 1. Research, inquiry and analytical thinking abilities, 4. Communication, 5. Teamwork, collaborative and management skills, 6. Information literacy

Teaching strategies underpinning the course

There are a number of key results in the subject that summarise the main discoveries of workers in the area. The statements and proofs of these results, which will be flagged during the lectures, must be learnt.

New ideas and skills are introduced and demonstrated in lectures, then you develop these skills by applying them to specific tasks in tutorials and assessments.
Rationale for learning and teaching strategies

We believe (and experience suggests) that effective learning is best supported by a climate of inquiry, in which you are actively engaged in the learning process. Hence this course is structured with a strong emphasis on problem-solving tasks in tutorials and in assessment tasks, and you are expected to devote much your class and study time to the solving of such problems.

Assessment

Assessment in this course will consist of three short assignments, each worth 10%, and a final examination, worth 70%.

Knowledge and abilities assessed: All assessment tasks will assess the learning outcomes outlined above, specifically, your ability to present logical and coherent mathematical arguments, and to solve mathematical problems using a variety of methods.

Assessment criteria: The main criteria for marking all assessment tasks will be clear and logical presentation of correct solutions; creativity will also be valued.

Assignments

Rationale: Assignments give an opportunity for you to try their hand at more difficult problems requiring more than one kind of argument and also introduce you to some aspects of the subject which are not explicitly covered in lectures.

The assignments will involve solving problems that are reasonably close to those presented in the course, and similar to those which will appear on the final examination; while students are encouraged to discuss the problems amongst themselves, the solutions should be written up individually.

Since not all students in the course may be used to presenting carefully written solutions to problems, there will also be a trial problem set, not for assessment but for feedback purposes only. Doing these and handing solutions in is optional, but strongly advised, especially for students who are not familiar with the evaluation criteria adopted in the School of Mathematics and Statistics.

Assessment Schedule:

<table>
<thead>
<tr>
<th>Task</th>
<th>Date Avail.</th>
<th>Date Due</th>
<th>Form of Submission</th>
<th>Weighting</th>
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</thead>
<tbody>
<tr>
<td>Trial problems</td>
<td>Week 1</td>
<td>Thur Week 2</td>
<td>Written</td>
<td>0%</td>
</tr>
<tr>
<td>Assignment 1</td>
<td>Week 2</td>
<td>Mon Week 4</td>
<td>Written</td>
<td>10%</td>
</tr>
<tr>
<td>Assignment 2</td>
<td>Week 5</td>
<td>Mon Week 7</td>
<td>Written</td>
<td>10%</td>
</tr>
<tr>
<td>Assignment 3</td>
<td>Week 8</td>
<td>Mon Week 10</td>
<td>Written</td>
<td>10%</td>
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Late assessments will be penalised by 10% per day of the available assessment mark. Every class is different, and to accommodate this, some variation from the above schedule may be prudent. Hence the above schedule should be considered as a guide only, as it may be varied slightly. In the case of assessment dates, no changes will be made without consultation with the class; confirmation of any changes will being posted on Blackboard.

Appropriate acknowledgement of the work of others in all assessed work that you do is essential. For more information, see the University web page on plagiarism: [www.lc.unsw.edu.au/plagiarism](http://www.lc.unsw.edu.au/plagiarism).

**Examination**

**Duration:** Two hours.

**Rationale:** The final examination will assess your mastery of the material covered in the lectures, and your ability to extend it to solve new problems.

**Weighting:** 70% of your final mark.

Further details about the final examination will be available in class closer to the time.

**Additional resources and support**

**Tutorial Exercises**

A set of tutorial exercises will be handed out in class. These problems are for you to do to enhance mastery of the course.

Some of the problems will be done in tutorials, but you will learn a lot more and be better prepared for the final examination if you try to do them before the tutorial.

**Textbooks**

There is no set text for this course. There are many textbooks introducing Functional Analysis in the University Library, in particular, the following may be useful:

- J. B. Conway: *A Course in Functional Analysis*.
- W. Rudin: *Functional Analysis*.
Lecture notes

Lecture notes will not be provided.

Blackboard

All course materials and handouts will be available on Blackboard. These will be revised, to reflect, for example, the lecture timetable once this is finalised.

You should check regularly for new and updated material.

Course Evaluation and Development

The School of Mathematics and Statistics evaluates each course each time it is run. We carefully consider your responses and their implications for course development. It is common practice to discuss informally with students in the course how the course and their mastery of it are progressing. Your feedback will be highly valued in order to improve it for future years.

Administrative matters

Additional Assessment

See attached handout.

School Rules and Regulations

Fuller details of the general rules regarding attendance, release of marks, special consideration etc are available via the School of Mathematics and Statistics Web page at http://www.maths.unsw.edu.au/students/current/policies/studentpolicy.html.

Plagiarism and academic honesty

Plagiarism is the presentation of the thoughts or work of another as one’s own. Issues you must be aware of regarding plagiarism and the university’s policies on academic honesty and plagiarism can be found at http://www.lc.unsw.edu.au/plagiarism and http://www.lc.unsw.edu.au/plagiarism/plagiarism_STUDENTBOOK.pdf.
Detailed course schedule

It is intended to cover the following topics, not necessarily exactly in the given order. Any variation from this will be indicated by the lecturer.

Chapter 1 – INTRODUCTION TO NORMED VECTOR SPACES
Definitions; interaction between topology and linear structure; isomorphism and isometry; convexity.

Chapter 2 – HILBERT SPACES
Definitions; projections, best approximations and Bessel’s Inequality; bases; linear operators on a Hilbert space; spectrum and resolvent; functions of operators; spectral theory of linear operators (compact operators only).

Chapter 3 – BANACH SPACES
Examples including $\ell^p$ spaces and Sobolev-type spaces; introduction to the geometry of Banach spaces; dual spaces; the Baire Category Theorem.

Chapter 4 – LINEAR OPERATORS
The Hahn–Banach, Open Mapping, Closed Graph and Banach-Steinhaus Theorems.