MATH5825 – Course Outline

Information about the course

Course Authority: Dr. H. Grundling

Lecturer: Dr. H. Grundling RC-5104, email h.grundling@unsw.edu.au.

Consultation: Times will be announced in week 2.

Credit & Prerequisites:
This course counts for 6 Units of Credit (6UOC).
MATH3611 is assumed knowledge for this course.

Lectures: There will be three lectures per week. The times will be decided at the timetabling meeting Monday 09h00, wk1.

Blackboard 9: Some low level activity may take place on the webmodule for MATH5825 in Blackboard 9.

Course aims

Measure theory provides a foundation for many branches of mathematics such as harmonic analysis, ergodic theory, theory of partial differential equations and probability theory. It is a central, extremely useful part of modern analysis, and many further interesting generalizations of measure theory have been developed. It is also subtle, with surprising, sometimes counterintuitive, results. The aim of this course is to learn the basic elements of Measure Theory.

Detailed course schedule

It is intended that the following topics will be covered in the given order.

1. Revision of Riemann integration in \( \mathbb{R}^n \), problems of the Riemann integral.
   Lebesgue’s “problem of measure” in \( \mathbb{R}^n \), Vitali set, Banach–Tarski theorem.
2. Abstract measure theory - \( \sigma \)-algebras, measurable sets, measures, outer measures, Lebesgue measure and its properties, completion of measures.
3. Measurable functions, approximation by simple functions.
Product measures, Tonelli–Fubini theorem.

Locally compact spaces, Radon measures, linear functionals, Riesz representation theorem.

$L^p$ spaces, Riesz–Fischer theorem, special spaces $L^\infty$ and $L^1$.

Signed measures, Hahn decomposition theorem, Jordan decompositions, absolute continuity of measures, Lebesgue decomposition, Radon–Nikodym Theorem, Radon–Nikodym derivatives, chain rule, image measures.

Disintegration of measures and conditional expectations.

(If time permits) Measures on infinite dimensional spaces, Gaussian measures.

Assessment

Assessment in this course will consist of two assignments (worth 15% each) and a 3 hour final examination (70%). Due dates for these are:

<table>
<thead>
<tr>
<th>Task</th>
<th>Date Due</th>
<th>Duration</th>
<th>Weighting</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ass 1</td>
<td>Fri Week 5</td>
<td>2 weeks</td>
<td>15%</td>
</tr>
<tr>
<td>Ass 2</td>
<td>Fri Week 11</td>
<td>2 weeks</td>
<td>15%</td>
</tr>
</tbody>
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Late assignments will not be accepted.

Examination

Duration: Three hours. Weighting: 70% of your final mark.

Further details about the final examination will be available in class closer to the time.

Additional resources and support

Problem Sheets

A set of problem sheets will be given out. These problems are for you to do to enhance mastery of the course.

Lecture notes

Some skeleton notes may be provided on the course website from time to time.
Textbooks

There is no set textbook for this course, but the following books will be helpful.

- D. L. Cohn: measure theory, Birkhauser 1993
- W. Rudin: Real and complex analysis P515/33
- Bartle, R.G. The elements of integration P517.52/29
- Billingsley, P. Probability and Measure, P519.1/492
- Halmos, P.R. Measure theory, P517.52/24
- Kelley, J.L., Measure and Integral, P517.52/173/(1)
- Weir, A.J. Lebesgue integration and measure, P517.52/69
- Munroe, M.E. Introduction to measure and integration, P517.52/90
- Doob, J.L. Measure theory, P517.52/171

Course Evaluation and Development

The School of Mathematics and Statistics evaluates each course each time it is run. We carefully consider the student responses and their implications for course development. It is common practice to discuss informally with students how the course and their mastery of it are progressing.

Student Learning Outcomes

Students taking this course will develop an appreciation of the basic concepts of measure theory. These methods will be useful for further study in a range of other fields, e.g. Quantum Theory, Stochastic calculus and Harmonic analysis.

Relation to graduate attributes: The above outcomes are related to the development of the Science Faculty Graduate Attributes, in particular: 1. Research, inquiry and analytical thinking abilities, 4. Communication, 6. Information literacy
Teaching strategies underpinning the course

New ideas and skills are introduced and demonstrated in lectures, then students develop these skills by applying them to specific tasks in problem sheets and assessments.

Rationale for learning and teaching strategies

We believe that effective learning is best supported by a climate of inquiry, in which students are actively engaged in the learning process. Hence this course is structured with a strong emphasis on problem-solving tasks in assessment tasks, and students are expected to devote the majority of their class and study time to the solving of such tasks.

Rationale for Assignments: Assignments will give an opportunity for students to try their hand at more difficult problems requiring more than one line of argument and also introduce them to aspects of the subject which are not explicitly covered in lectures.

Rationale for Examinations: The final examination will assess student mastery of the material covered in the lectures.

Assessment criteria: The main criteria for marking all assessment tasks will be clear and logical presentation of correct solutions.

Knowledge and abilities assessed: All assessment tasks will assess the learning outcomes outlined above.

Administrative matters

Additional Assessment

This is at
http://www.maths.unsw.edu.au/students/current/policies/addasspolicy.html

School Rules and Regulations

Fuller details of the general rules regarding attendance, release of marks, special consideration etc are available via the School of Mathematics and Statistics Web page at http://www.maths.unsw.edu.au/students/current/policies/studentpolicy.html.
Plagiarism and academic honesty

Assignments must be YOUR OWN WORK, or severe penalties will be incurred. You should consult the University web page on plagiarism. Plagiarism is the presentation of the thoughts or work of another as one’s own. Issues you must be aware of regarding plagiarism and the university’s policies on academic honesty and plagiarism can be found at http://www.lc.unsw.edu.au/plagiarism and http://www.lc.unsw.edu.au/plagiarism/plagiarism_STUDENTBOOK.pdf.