

MATHEMATICS ENRICHMENT CLUB.¹

Problem Sheet 13, August 20, 2012

1. Laurie sold two cars for \$25 000 each. One he sold at a 20% profit and the other at a 20% loss. How much did he gain or lose ?
2. A pizza has radius z , and height a . What is interesting about it's volume?²
3. If a triangle ABC has sides of length a, b, c such that $a^2 + b^2 = c^2$, prove that it must be a right-angled triangle.
4. Without using a calculator, which is larger 31^{24} or 257^{15} .
5. Let $S_n = 2n(2n - 1)(2n - 2)\dots(n + 1)$. For example, $S_3 = 6 \times 5 \times 4 = 120$.
 - (a) What is the power of 2 in the prime factorisatoin of S_n for $n = 2, 3, 4, \dots$?
 - (b) Make a conjecture based on(i) and prove it.
6. Without using a calculator, show that

$$\sqrt[3]{5\sqrt{13} + 18} - \sqrt[3]{5\sqrt{13} - 18} = 3.$$

(Hint: Let $x = a - b$ and cube.)

7. Let ABC be a triangle and D, E points on AB, BC respectively, and S be the intersection of AE and CD . If $AD = DB$ and $BE : EC = 2 : 1$, find the ratios $CS : SD$ and $AS : SE$.
8. (a) Let P be an interior point in an equilateral triangle ABC . Prove that we can always form a triangle with sides of length AP, BP, CP . (That is, we have to show that the sum of any two of these lengths is larger than the remaining one.)
 - (b) Give an example of a triangle and point inside it for which the above result is not true.

¹Some of the problems here come from T. Gagen, Uni. of Syd. and from E. Szekeres , Macquarie Uni.

²This question thanks to Mike Hirschhorn