MATHEMATICS ENRICHMENT CLUB.\footnote{Some of the problems here come from T. Gagen, Uni. of Syd. and from E. Szekeres, Macquarie Uni.}
Problem Sheet 5, June 4, 2013

1. Express the number 0.504504504... as a fraction in lowest terms.

2. Yvonne and Znonimir play a game. They have a pile of 500 counters and each is allowed to remove 1, 2, 4, 8,... counters from the pile, taking turns in so doing. The last person to take a counter loses. Assuming that both play using the best possible strategy at each go, who wins?

3. The last digit of $1997^{1997}$ is
   \[ (a) \ 1 \quad (b) \ 3 \quad (c) \ 5 \quad (d) \ 7 \quad (e) \ 9. \]

4. How many planes of symmetry has a rectangular box of dimensions $2 \times 2 \times 3$?

5. (a) Paul measured all 6 edges of a tetrahedron $ABCD$ and found them to be 1, 3, 4, 5, 6, 8 cm. Can this be correct?
   (b) Paul then measured the edges to be 2, 3, 4, 5, 6, 8. If $AB = 2$ what is the length of $CD$?

6. (a) Prove that the angle in a semicircle is right-angle.
   (b) Show that if two chords of a circle mutually bisect each other, then they are both diameters.
   (c) Complete the following statement: If a parallelogram is inscribed in a circle then ........

Senior Questions

1. Put $x = z - \frac{1}{z}$ and hence solve, in surd form, the cubic $x^3 + 3x = 1$.

2. Let $ABC$ be a triangle with three medians intersecting at $S$. Let $L, M$ be the midpoints of $AC$, $AB$ respectively.
   (a) Prove that the triangles $LSC$ and $MSB$ have equal areas.
   (b) Given that $LSC$ has area $100\text{cm}^2$, find the area of $ABC$. 