1. Which is larger $100^{300}$ or $300!$?

2. A chess board is an $8 \times 8$ grid of squares coloured white or black so that no two adjacent squares are the same colour. Given tiles that are $2 \times 1$ grid squares it is possible to cover the chessboard completely, and it takes precisely 32 tiles. Show that it is impossible to cover a chessboard with opposite corners removed.

3. Find all 3 digit numbers which are equal to the sum of the factorials of their digits.

4. The triangle $\Delta ABC$ is right with hypotenuse $AB$. Two semi-circles are drawn on the exterior of the triangle with diameters $AC$ and $BC$. The semi-circle through $ABC$ is also drawn. Compute the sum of the areas bounded by the two arcs which meet at $A$ and $C$, and the two arcs which meet at $B$ and $C$.

5. Tic-tac-toe is a game played by two players who take turns marking either $x$ or $o$ in a square on a $3 \times 3$ grid. A player wins if they get 3 of their symbols in a row, but if the grid is filled without a winner the game is a draw. How many tic-tac-toe games end in a draw?

6. (a) Let $p$ denote the perimeter of a triangle and $r$ the radius of its inscribed circle. Show that $r \leq \frac{p}{6\sqrt{3}}$.

(b) $P_1, P_2, \ldots, P_n$ are the vertices of an $n$-gon inscribed in a circle. The centre, $O$, of the circle lies inside the $n$-gon. Let $A$ denote the sum of the areas of the circles inscribed in the $n$ triangles $\Delta P_1OP_2$, $\Delta P_2OP_3$, $\ldots$, $\Delta P_nOP_1$, and let $B$ denote the area of the $n$-gon. Show that $\frac{A}{B} \leq \frac{\pi}{3\sqrt{3}}$. If $0 < k < \frac{\pi}{3\sqrt{3}}$ show that for any $n \geq 3$ there exists a polygon with $k = \frac{A}{B}$.

Senior Questions

1. Let $f$ and $g$ be real valued, continuous functions defined for $-1 \leq x \leq 1$. Denote by

$$\langle f, g \rangle = \left( \int_{-1}^{1} (f(x)g(x))^2 \, dx \right)^{\frac{1}{2}}.$$
Two functions, $f$ and $g$, are said to be orthogonal if $\langle f, g \rangle = 0$. Let $p_0(x) = 1$ and $p_1(x) = x$,

(a) Show that $p_0$ and $p_1$ are orthogonal.

(b) Find $p_2(x)$, a degree 2 polynomial, which is orthogonal to both $p_0$ and $p_1$.

(c) Show that any polynomial of degree less than or equal to 2 can be written as a linear combination of $p_0$, $p_1$ and $p_2$, i.e. for any polynomial $g$ with $\deg g \leq 2$, there exists constants $\alpha_0$, $\alpha_1$ and $\alpha_2$ such that

$$g(x) = \alpha_0 p_0(x) + \alpha_1 p_1(x) + \alpha_2 p_2(x).$$