1. Compute the product
\[
\left(1 - \frac{1}{2^2}\right) \left(1 - \frac{1}{3^2}\right) \cdots \left(1 - \frac{1}{(1+k)^2}\right) \cdots \left(1 - \frac{1}{2014^2}\right)
\]

2. A 10 digit number is beautiful if it has the digits 0, 1, 2, 3, 4 appear in order and 9, 8, 7, 6, 5 appear in order. For example 9081726345 (9081726345) is beautiful. If they can’t start with 0, how many 10 digit beautiful numbers are there?

3. The numbers on opposite faces of a standard 6 sided die all add up to 7. Suppose I have 27 standard dice arranged in a $3 \times 3 \times 3$ cube. What is the smallest possible sum of all the exposed faces?

4. The incircle of a triangle $\triangle ABC$ is centred at $O$ and meets $AC$ at $P$ and $BC$ at $Q$. The line $BO$ is extended to meet $PQ$ at $G$. Show that $\triangle AOG$ is right angled.

5. Two people agree to meet for coffee some time between 10am and 11am but don’t specify an exact time. They each arrive at a random time between 10am and 11am and wait for 15 minutes. What is the probability they see each other?

6. Ron chooses a point on a chessboard. Hermione can draw any polygon (without self intersections) on the chessboard and ask whether Ron’s point is inside or outside her polygon. What is the smallest number of questions Hermione can ask to determine whether Ron’s point is in a black or white square?

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1Some problems from UNSW’s publication Parabola
Senior Questions
Some basic set theory: Let’s called \( \mathbb{R} \) “the set of all real numbers”, so if \( x \in \mathbb{R} \) (said as “\( x \) is in \( \mathbb{R} \)” we mean \( x \) is a real number. We can write other sets of real numbers that satisfy a formula like this: \( A = \{ x \in \mathbb{R} : F(x) \} \) read as “\( A \) is the set of all real numbers, \( x \), which satisfy \( F(x) \)”. For instance \( A = \{ x \in \mathbb{R} : x^2 = 4 \} \) is the set \( \{-2, 2\} \).

- The “union” of two sets, \( A \cup B \) is a new set which contains all of the elements of \( A \) and the elements of \( B \), i.e. \( A \cup B = \{ x \in \mathbb{R} : x \in A \text{ or } x \in B \} \).

- The “intersection” of two sets, \( A \cap B \) is a new set which contains all of the elements that are in \( A \) and \( B \) at the same time, i.e. \( A \cap B = \{ x \in \mathbb{R} : x \in A \text{ and } x \in B \} \).

- The “complement” of a set, \( A^c \), is the set of all numbers not in \( A \), i.e. \( A^c = \{ x \in \mathbb{R} : x \notin A \} \).

1. Show that \( (A \cup B)^c = A^c \cap B^c \).

2. Show that \( (A \cap B)^c = A^c \cup B^c \).