Solution Sheet 16, September 10, 2012

Answers

1. 3 boxes of apples, 1 box of oranges, 4 boxes of lemons.

2. Find the area of the larger square in two different ways.

3. Let the rectangle have sides of length \(a, b\) where

\[
a + b = 10. \tag{1}
\]

By Pythagoras’ theorem, the diagonal \(d\) has length

\[
d^2 = a^2 + b^2 \tag{2}
\]

Sub equation (1) into equation 2, the result is

\[
d^2 = 2a^2 - 20a + 100
\]

Use the formula for the minimum value of a parabola to find that \(a = b = 5\). Hence \(d = 2\sqrt{5}\).

4. \(y_n = 1 \mod 8\) but \(x_2 = 3 \mod 3\) and \(x_3 = 5 \mod 8\). For larger values of \(n\):
   \(x_n = 3 + 2 \times 5 \mod 8 = 5 \mod 8\) or \(x_n = 5 + 2 \times 3 \mod 8 = 5 \mod 8\)

5. Using \(a^2 + b^2 \geq 2ab\),

\[
\begin{align*}
2(a^2 + b^2 + c^2) & \geq 2(ab + bc + ca) \\
3(a^2 + b^2 + c^2) & \geq (a^2 + b^2 + c^2) + 2(ab + bc + ca) \\
3(a^2 + b^2 + c^2) & \geq (a + b + c)^2 \\
a^2 + b^2 + c^2 & \geq \left(\frac{a + b + c}{3}\right)^2
\end{align*}
\]