



UNSW
THE UNIVERSITY OF NEW SOUTH WALES

FACULTY OF SCIENCE
SCHOOL OF MATHEMATICS
AND STATISTICS

MATH3201
DYNAMICAL SYSTEMS
AND CHAOS

Session 1, 2009



MATH3201 – Course Outline

Information about the course

Course Authority: A/Prof. G. Froyland

Lecturer: A/Prof. G. Froyland RC-2083, email G.Froyland@unsw.edu.au.

Consultation: Please use email to arrange an appointment.

Credit, Prerequisites:

This course counts for 6 Units of Credit (6UOC).

Prerequisites: 12 units of credit in Level 2 Mathematics courses including MATH2120 or MATH2130, and MATH2501 or MATH2601, or both MATH2019(DN) and MATH2089, or both MATH2069(CR) and MATH2099.

Lectures: There will be three lectures per week:

Wednesday 2–4pm	RC4082
Thursday 1pm	RC4082

Tutorials: There will be one tutorial per week:

Thursday 3pm	RC4082 (for Theory Tutorials) or
Thursday 3pm	Red Centre Windows Lab – RC-G012C (for Computer Labs)

Tutorials will approximately alternate between theory and computer lab sessions. The first tutorial in week 1 will be a theory tutorial. Computer lab tutorials will be held in the Red Centre computing labs; more information will be provided in lectures.

WebCT: Further information, and other course material will be provided via WebCT.

Course Description

Many nonlinear ODEs do not have explicit solutions. The dynamical systems approach shifts the focus from finding explicit solutions to discovering geometric properties of solutions. It also recognises that even a small amount of nonlinearity in a physical system can be responsible for very complicated chaotic behaviour. In this course you will learn the fundamentals of dynamical systems in (continuous time) nonlinear ODEs and in (discrete time) nonlinear maps, allowing you to analyse the local and global behaviour of nonlinear systems. You will also learn how to analyse time series data using nonlinear tools and build appropriate predictive models. Applications and computations play a significant role in this course and MATLAB will be used frequently.

1. Nonlinear maps: The building blocks of dynamics

fixed points, periodic points, invariant sets, recurrence, nonwandering sets, symbolic dynamics, conjugacy, sensitive dependence on initial conditions, Lyapunov exponents, fractals, Stable Manifold Theorem, chaotic attractors, Smale's Horseshoe, ergodic theory

2. Nonlinear ODEs: A geometric, qualitative approach to ODEs

phase portraits, fixed points, periodic and chaotic trajectories, sources, sinks, and saddles, stable and unstable subspaces, robustness, hyperbolicity, stability, conjugacy, stable and unstable manifolds, bifurcations

3. Nonlinear time series analysis: What to do with real data?

stationarity, linear or nonlinear, embedding and delay reconstruction, invariants, modelling and forecasting

Relation to other mathematics courses and other disciplines

Dynamical Systems is a subject that sits at the threshold of pure and applied mathematics and has links to many other areas of mathematics, including Analysis, Linear Algebra, Measure Theory, Ergodic Theory, Functional Analysis, Topology, Numerical Analysis, Stochastic Processes, Group Theory, and Mathematical Modelling.

This course will make use of many pure mathematical tools that you have learnt so far, and refine those parts of pure mathematics that are particularly useful for studying dynamical systems. The numerical component will make use of many applied mathematical methods you already know and develop more specialised methods.

This course is very useful for those majoring in Applied or Pure Mathematics, those interested in being able to model and understand dynamical phenomena (eg. stock markets, the weather, biological populations) at a deeper level, and those planning to teach.

Dynamical Systems has applications in Engineering, Physics, Chemistry, Space, Biology, and Computer Science and those majoring in these disciplines would also benefit from the course.

Student Learning Outcomes

Students taking this course will develop an appreciation of the usefulness of the mathematics that they have learned so far and the connection between dynamical systems and other mathematics subjects. Students will develop teamwork skills via the team-based project assessment. An emphasis will be upon problem solving and students will develop and hone their problem solving skills via tutorial questions and the team projects.

Through regularly attending lectures and applying themselves in tutorial exercises, students will develop competency in mathematical presentation, written and verbal skills.

Relation to graduate attributes

The above outcomes are related to the development of the Science Faculty Graduate Attributes, in particular: **1. Research, inquiry and analytical thinking abilities**, **4. Communication**, **6. Information literacy**

Teaching strategies underpinning the course

New ideas and skills are introduced and demonstrated in lectures, then students develop these skills by applying them to specific tasks in tutorials and assessments.

Computing skills are developed and practised in computer practical sessions.

Assessment in this course will use problem-solving tasks of a similar form to those practiced in tutorials, to encourage the development of the core analytical and computing skills underpinning this course.

Rationale for learning and teaching strategies

We believe that effective learning is best supported by a climate of enquiry, in which students are actively engaged in the learning process. To ensure effective learning, students should participate in class as outlined below.

We believe that effective learning is achieved when students attend all classes, have prepared effectively for classes by reading through previous lecture notes, in the case

of lectures, and, in the case of tutorials, by having made a serious attempt at doing for themselves the tutorial problems prior to the tutorials.

Furthermore, lectures should be viewed by the student as an opportunity to learn, rather than just copy down lecture notes.

Effective learning is achieved when students have a genuine interest in the subject and make a serious effort to master the basic material.

The art of logically setting out mathematics is best learned by watching an expert and paying particular attention to detail. This skill is best learned by regularly attending classes.

Assessment

Assessment in this course will consist of two team projects (40% in total) and a final examination (60%).

Knowledge and abilities assessed: All assessment tasks will assess the learning outcomes outlined above, in particular, the understanding of dynamical systems concepts, the ability to prove theoretical results, the ability to solve problems via dynamical systems methods, both theoretically and numerically by computer.

Assessment criteria: The main criteria for marking all assessment tasks will be clear and logical presentation of correct solutions.

Assignments

Rationale: Team projects will give an opportunity for students to try their hand at more difficult problems requiring more than one line of argument and also introduce them to aspects of the subject which are not explicitly covered in lectures.

Team projects must include an EQUAL CONTRIBUTION FROM ALL TEAM MEMBERS, or severe penalties will be incurred.

Please refer to the section on plagiarism later in this document.

The following table is an estimate of the team project timing and weightings. Minor variations may occur.

Task	Date Avail.	Date Due	Form of Submission	Weighting
Proj 1	Thurs Week 3	Fri Week 5	Written	15%
Proj 2	Thurs Week 7	Fri Week 11	Written	25%

Late assignments will not be accepted.

Examination

Duration: Two hours.

Rationale: The final examination will assess student mastery of the material covered in the lectures.

Weighting: 60% of your final mark.

Further details about the final examination will be available in class closer to the time.

Additional resources and support

Tutorial Exercises

A set of tutorial exercises will be given out. These problems are for YOU to do to enhance mastery of the course.

SOME of the problems will be done in tutorials, but you will learn a lot more if you try to do them before the tutorial.

Lecture notes

Photocopies will be available after lectures at the School Office on Level 3.

Textbooks

There is no set text for this course.

The content of the course will be defined by the lectures. Any book on Dynamical Systems may prove useful.

myeLearning Vista

Most course materials will be available on myeLearning Vista. You should check regularly for new materials.

Course Evaluation and Development

The School of Mathematics and Statistics evaluates each course each time it is run. We carefully consider the student responses and their implications for course development. It is common practice to discuss informally with students how the course and their mastery of it are progressing.

Administrative matters

Additional Assessment

See attached handout.

School Rules and Regulations

Students must read and understand the School of Mathematics and Statistics Policies as contained in the 'Important Information for Students' document. This can be found on the web at

http://www.maths.unsw.edu.au/students/current/policies/UG_student_info.pdf
and is appended for your convenience.”

Plagiarism and academic honesty

Plagiarism is the presentation of the thoughts or work of another as one's own. Issues you must be aware of regarding plagiarism and the university's policies on academic honesty and plagiarism can be found at <http://www.lc.unsw.edu.au/plagiarism/index.html> and <http://www.maths.unsw.edu.au/students/current/policies/misconductpolicy.html>.

Detailed Course Schedule

Local and Global behaviour of nonlinear maps in \mathbb{R}

- Fixed points, periodic points, homeomorphisms, diffeomorphisms, orbits.
- Stability, instability, stable and unstable sets.
- Graphical analysis to find fixed points and stable and unstable sets.
- Derivative condition for stable periodic orbits.
- Saddle node (tangent) bifurcation, period doubling bifurcation.
- ω -limit sets, invariant sets, recurrent points.
- Transitivity
- Conjugacy between smooth maps.
- Sensitive dependence on initial conditions.
- Chaos = sensitive dependence + transitivity (and both are conjugacy invariants).
- Lyapunov exponents (quantifying chaos).
- Invariant measures, ergodic invariant measures, Birkhoff Ergodic Theorem, long term distribution of orbits.

Global behaviour of linear maps in \mathbb{R}^n

- Sources, sinks and saddles.
- Conjugacy of sources, conjugacy of sinks.
- Hyperbolic linear maps.
- Stable, unstable and centre subspaces; characterisation as eigenspaces.
- Robustness of hyperbolic linear maps and robustness of E^u, E^c, E^s .

Local behaviour of nonlinear maps in \mathbb{R}^n

- Derivative condition for stable periodic orbits.
- Definition of E_p^u, E_p^s, E_p^c for fixed point p .
- Global dynamics of Hénon map; local dynamics of Hénon map about a fixed point.

- Hartman-Grobman Theorem (local conjugacy about p between $Df(p)$ and f).
- Stable manifolds, Stable Manifold Theorem (stable set is a manifold and tangency to E_p^s at p).
- Inclination Lemma (near saddle points forward iterates of line segments approach unstable manifold of saddle point).

Global behaviour of nonlinear maps in \mathbb{R}^n

- Global stable manifold by iterating local stable manifold.
- Trapping region, attracting set, attractor, chaotic attractor.

Global behaviour of nonlinear flows in \mathbb{R}^n

- Flow, vector field, phase portrait.

Global behaviour of linear flows in \mathbb{R}^n

- Flow, vector field, phase portrait.
- Definition of saddle, node, centre, focus, and closed form solutions for trajectories.
- Eigenvalue condition for stability of the fixed point at the origin.
- Stable, centre, unstable subspaces for linear flows; characterisation in terms of eigenspaces.
- Robustness of hyperbolic flows (conjugacy and continuity of E^u, E^s).

Existence and simulation of nonlinear flows in \mathbb{R}^n

- Numerical solutions of ODEs, Euler's method, explicit midpoint rule, trapezoidal rule.
- Order of a numerical method.
- Existence and uniqueness of solutions of ODEs.

Local behaviour of nonlinear flows in \mathbb{R}^n

- Definition of linearised DE about a fixed point p ; (HGT tell us later that local nonlinear dynamics about p are conjugate to the linearised DE dynamics).
- Damped pendulum: Identification of local behaviour about fixed points, study of phase portraits, stable manifolds of fixed points.
- Derivative condition for local stability of fixed points.
- Hartman-Grobman Theorem for flows (nonlinear system locally conjugate to linearised system about a fixed point).

Global behaviour of nonlinear flows in \mathbb{R}^2

- Periodic orbits, ω -limit points and ω -limit sets.
- Poincaré-Bendixson Theorem.
- Poincaré map.
- Method of nullclines.
- Predator-prey example: Identification of nullclines, subdividing phase portrait into regions of common movement, finding ω -limit sets for different initial conditions.

Nonlinear time series analysis

- Dynamical system and observation function formalism.
- Stationarity, testing for stationarity.
- Good models are simple models.
- Linear+noise (AR) models, advantages and disadvantages.
- Nonlinear deterministic models, Takens' Embedding Theorem.
- Building a nonlinear model, choice of embedding dimension, lags, ball size.
- Verification of model quality, short term prediction vs. long term behaviour.