THE UNIVERSITY OF NEW SOUTH WALES

FACULTY OF SCIENCE

SCHOOL OF MATHEMATICS AND STATISTICS

MATH5975

INTRODUCTION TO STOCHASTIC ANALYSIS

Session 1, 2010

MATH5975 – Course Outline

Information about the course

Course Authority: Dr Donna Mary Salopek

Lecturer: Dr Donna Mary Salopek RC-2051,
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Consultation: Please use email to arrange an appointment.

Credit: This course counts for 6 Units of Credit (6UOC).

Lectures: Fridays 5.00-8.00 pm in RC 4082

Blackboard: Lecture notes and other material will be provided via Blackboard.

Course aims

Modern theory of financial markets relies on advanced mathematical and statistical methods that are used to model, forecast and manage risk in complex financial transactions. After the publication in 1973 of the ground-breaking paper of Black and Scholes on the arbitrage pricing of European call options, it became clear that Stochastic Analysis is an indispensable tool for the theory of financial markets, derivation of prices of standard and exotic options and other derivative securities, hedging related financial risk, as well as managing the interest rate risk.

In this course, you will learn the basic concepts and techniques of Stochastic Analysis, such as: Brownian motion, martingales, Itô stochastic integral, Itô's formula, stochastic differential equations, equivalent change of a probability measure, integral representation of martingales with respect to a Brownian filtration, relations to second order partial differential equations, and the Feynman-Kac formula.

Some concepts will be illustrated by examples relevant for financial applications. However, the main goal of the course is to provide a necessary mathematical background for MATH5816 Continuous Time Financial Modelling and MATH5985 Term Structure Modelling, rather then to focus directly on financial concepts.

Relation to other mathematics courses

The course is a prerequisite for MATH5816 Continuous Time Financial Modelling and MATH5985 Term Structure Modelling and will provide a solid background for your Master Project as well.

Student Learning Outcomes

Students taking this course will develop an appreciation of the basic problems of stochastic analysis and be able to

- Recognise which analysis procedure is appropriate for a given research problem
- Apply probability theory and stochastic analyis to practical problems
- Understand the usefulness of Stochastic Analysis in your professional area.

The ability to provide logical and coherent proofs of theoretic results, and the ability to solve problems via abstract methods will be paramount.

Through regularly attending lectures and applying themselves, students will develop competency in mathematical presentation, written and verbal skills.

Relation to graduate attributes

The above outcomes are related to the development of the Science Faculty Graduate Attributes, in particular: 1.**Research, inquiry and analytical thinking abilities**, 2. **Communication**, 3. **Information literacy**

Teaching strategies underpinning the course

New ideas and skills are introduced and demonstrated in lectures, then students develop these skills by applying them to specific tasks in tutorials and assessments.

Rationale for learning and teaching strategies

We believe that effective learning is best supported by a climate of enquiry, in which students are actively engaged in the learning process. To ensure effective learning, students should participate in class as outlined below.

We believe that effective learning is achieved when students attend all classes, have prepared effectively for classes by reading through previous lecture notes, in the case of lectures, and, in the case of tutorials, by having made a serious attempt at doing for themselves the tutorial problems prior to the tutorials.

Furthermore, lectures should be viewed by the student as an opportunity to learn, rather than just copy down lecture notes.

Effective learning is achieved when students have a genuine interest in the subject and make a serious effort to master the basic material.

The art of logically setting out mathematics is best learned by watching an expert and paying particular attention to detail. This skill is best learned by regularly attending classes.

Assessment

Assessment in this course will consist of three assignments (30% each) and a final examination (70%). **Note: you will need to pass the final examination to pass the course**.

Knowledge and abilities assessed: All assessment tasks will assess the learning outcomes outlined above, specifically, the ability to solve mathematical problems.

Assessment criteria: The main criteria for marking all assessment tasks will be clear and logical presentation of correct solutions.

Assignments

Rationale: Assignments will give an opportunity for students to try their hand at more difficult problems requiring more than one line of argument and also introduce them to aspects of the subject which are not explicitly covered in lectures.

Assignments must be YOUR OWN WORK, or severe penalties will be incurred.

You should consult the University web page on plagiarism: `www.lc.unsw.edu.au/plagiarism`

Late assignments will not be accepted (unless the delay is justified by serious reasons).

Examination

Duration: Two hours.

Rationale: The final examination will assess student mastery of the material covered in the lectures. You will need to pass the final examination to pass the course.

Weighting: 70% of your final mark.

Further details about the final examination will be available in class closer to the time.

Additional resources and support

Lecture notes

A set of notes will be provided on Blackboard.

Textbooks

Suggested Readings:

S. Shreve, Stochastic Calculus for Finance II, Continuous Time Models, Springer 2004.

M. Capiński and T. Zastawniak: *Mathematics for Finance: An Introduction to Financial Engineering.* Springer, 2003.

F. Klebaner: *Introduction to Stochastic Calculus with Applications.* Imperial College Press, 2005.

Thomas Mikosch: *Elementary Stochastic Calculus with Finance in View.* World Scientific, Singapore, 1999.

A. Etheridge: *A Course in Financial Calculus.* Cambridge University Press, 2002.

Robert J. Elliott and P.E. Kopp: *Mathematics of Financial Markets.* Springer, Berlin Heidelberg New York, 1999.

Damien Lamberton and Bernard Lapeyre: *Introduction to Stochastic Calculus Applied to Finance.* Chapman and Hall, London, 1996.

Ioannis Karatzas and Stephen Shreve: *Brownian Motion and Stochastic Calculus.* Springer, Berlin Heidelberg New York, 1988.

Blackboard

All course materials will be available on Blackboard. You should check regularly for new materials.

Course Evaluation and Development

The School of Mathematics and Statistics evaluates each course each time it is run. We carefully consider the student responses and their implications for course development. It is common practice to discuss informally with students how the course and their mastery of it are progressing.

Administrative matters

Additional Assessment

See attached handout.

School Rules and Regulations

Fuller details of the general rules regarding attendance, release of marks, special consideration, etc., are available via the School of Mathematics and Statistics Web page at
`http://www.maths.unsw.edu.au/students/current/policies/studentpolicy.html`.

Plagiarism and academic honesty

Plagiarism is the presentation of the thoughts or work of another as one's own. Issues you must be aware of regarding plagiarism and the university's policies on academic honesty and plagiarism can be found at `http://www.lc.unsw.edu.au/plagiarism` and `http://www.lc.unsw.edu.au/plagiarism/plagiarism_STUDENTBOOK.pdf`.

Detailed course schedule

1. Conditional Expectation

2. Filtrations and Adapted Processes

3. Martingales

4. Standard Brownian Motion

5. Stopping Times and Martingales

6. Itô Stochastic Integral

   6.1. Itô Integral for Elementary Processes

   6.2. Itô Integral for Processes from $\mathcal{L}^2_{\mathbb{P}}(W)$

   6.3. Itô Integral for Processes from $\mathcal{L}_{\mathbb{P}}(W)$

7. Continuous Local Martingales

8. Continuous Semimartingales and Itô Processes

9. Itô's Lemma

   9.1. One-dimensional Case

   9.2. Multi-dimensional Case

   9.3. Itô-Tanaka-Meyer Formula

10. Lévy's Characterization Theorem

11. Martingale Representation Property

12. Stochastic Differential Equations

    12.1. Linear Stochastic Differential Equation

    12.2. Itô's Existence and Uniqueness Theorem for SDEs

13. Stochastic Exponential

14. Radon-Nikodým Density

15. Girsanov's Theorem

17. Feynman-Kac Formula