SCHOOL OF MATHEMATICS AND STATISTICS

MATH3201

DYNAMICAL SYSTEMS AND CHAOS

Term 3, 2021

1 MATH3201 – Course Outline

2 Information about the course

Course Authority: Prof. G. Froyland

Lecturer: Prof. G. Froyland RC3060, email `G.Froyland@unsw.edu.au`.

Consultation: Please use email to arrange an appointment.

Credit, Prerequisites: This course counts for 6 Units of Credit (6UOC). Prerequisites: (MATH2501 or MATH2601 or MATH2089 or MATH2099) and (MATH2011 or MATH2111 or MATH2018 (DN) or MATH2019 (DN) or MATH2069 (CR) or MATH2121 or MATH2221).

Lectures: There will be four lectures per week, except in week 6 (no lectures).

Wednesday 4–6pm	online
Thursday 2–4pm	online

Due to the current COVID restrictions, we will be moving the face-to-face Lecture online from week 1 until further notice. Hopefully, we can return to some In Person teaching from Week 7 or 8, but this will be assessed closer to the time. All Online Lectures will be on Blackboard Collaborate. You will be able to find the link for the Lecture by following the Live Lectures link on Moodle.

Tutorials: There will be one tutorial / computer lab per week:

Friday 3pm	online (for both theory tutorials and computer labs)

Tutorials will approximately alternate between theory and computer lab sessions. The first tutorial in week 1 will be a theory tutorial.

Moodle: Further information and other course material will be provided via Moodle.

3 Course Description

Many nonlinear systems do not have explicit solutions. The dynamical systems approach shifts the focus from finding explicit solutions to discovering geometric properties of solutions. It also recognises that even a small amount of nonlinearity in a physical system can be responsible for very complicated chaotic behaviour. In this course you will learn the fundamentals of dynamical systems in discrete-time maps and continuous-time ODEs, allowing you to analyse the local and global behaviour of dynamical systems. You will also learn how to analyse time series data using nonlinear tools and build appropriate predictive models. Throughout the course, MATLAB will be used frequently for computations.

1. **Nonlinear maps: The building blocks of dynamics**

   fixed and periodic points, stable/unstable sets, transitivity, conjugacy, sensitive dependence on initial conditions, Lyapunov exponents, invariant measures, recurrence, hyperbolicity, stable manifolds, chaotic attractors, Smale's Horseshoe.

2. **Nonlinear ODEs: A geometric, qualitative approach to ODEs**

   phase portraits, equilibria, periodic and chaotic trajectories, sources, sinks, and saddles, local stable and unstable subspaces, local hyperbolicity, Poincaré Bendixson.

3. **Nonlinear time series analysis: What to do with real data?**

   stationarity, linear or nonlinear, Takens embedding and delay reconstruction, conjugacy invariants, modelling and forecasting.

3.0.1 Relation to other mathematics courses and other disciplines

Dynamical Systems is a subject that sits at the threshold of pure and applied mathematics and has links to many other areas of mathematics, including Analysis, Linear Algebra, Measure Theory, Ergodic Theory, Functional Analysis, Topology, Numerical Analysis, Stochastic Processes, Group Theory, and Mathematical Modelling. This course will make use of many mathematical concepts and tools that you have learnt so far, and refine those parts of mathematics that are particularly useful for studying dynamical systems.

This course is very useful for those majoring in Applied or Pure Mathematics, and related disciplines, and those interested in being able to model and understand dynamical phenomena (e.g. weather and climate, fluid dynamics, molecular dynamics and drug design, viral infection dynamics, chemical reaction dynamics, population dynamics, celestial mechanics and space mission design) at a deeper level. Dynamical Systems has applications in Engineering, Physics, Chemistry, Space, Biology, and Computer Science and those majoring in these fields would also benefit from the course.

4 Student Learning Outcomes

Students taking this course will develop an appreciation of the usefulness of the mathematics that they have learned so far and the connection between dynamical systems and other mathematics subjects. Students will develop teamwork skills and group-based problem solving via team-based project assessment. Individual problem solving will be emphasised and students will develop and hone their problem-solving skills via tutorial questions. Through regularly attending lectures and applying themselves in tutorial exercises, students will develop competency in mathematical presentation, written and verbal skills.

4.0.1 Relation to graduate attributes

The above outcomes are related to the development of the Science Faculty Graduate Attributes, in particular: 1.**Research, inquiry and analytical thinking abilities**, 4. **Communication**, 6. **Information literacy**

5 Teaching strategies underpinning the course

New ideas and skills are introduced and demonstrated in lectures, then students develop these skills by applying them to specific tasks in tutorials and assessments. Computing skills are developed and practised in computer practical sessions. Assessment in this course will use problem-solving tasks of a similar form to those practiced in tutorials, to encourage the development of the core analytical and computing skills underpinning this course.

5.0.1 Rationale for learning and teaching strategies

We believe that effective learning is best supported by a climate of enquiry, in which students are actively engaged in the learning process. To ensure effective learning, students should participate in class as outlined below. We believe that effective learning is achieved when students attend all classes, have prepared effectively for classes by reading through previous lecture notes, in the case of lectures, and, in the case of tutorials, by having made a serious attempt at doing for themselves the tutorial problems prior to the tutorials. Furthermore, lectures should be viewed by the student as an opportunity to learn, rather than just copy down lecture notes. Effective learning is achieved when students have a genuine interest in the subject and make a serious effort to master the basic material. The art of logically setting out mathematics is best learned by watching an expert and paying particular attention to detail. This skill is best learned by regularly attending classes.

6 Assessment

Assessment in this course will consist of tests, assignments, or small-team projects (40% in total) and a final examination (60%). The form of the assessments may depend on the COVID-19 situation at the time.

Knowledge and abilities assessed: All assessment tasks will assess the learning outcomes outlined above, in particular, the understanding of dynamical systems concepts, the ability to prove theoretical results, the ability to solve problems via dynamical systems methods, both theoretically and numerically by computer.

Assessment criteria: The main criterion for marking all assessment tasks will be clear and logical presentation of correct solutions.

Late submission of assessment tasks: No late submissions will be accepted, where "late" in this context means after any extensions granted for Special Consideration or Equitable Learning Provisions.

6.1 In-term assessments

There will be a short formative assessment early in the course to provide feedback on how you are progressing. Later in the course there will be two summative assessments, likely to be small-team projects with some individual components.

Rationale: Team projects will give an opportunity for students to tackle difficult problems requiring more than one line of argument and introduce them to aspects of the subject that are not explicitly covered in lectures. Team projects must include an EQUAL CONTRIBUTION FROM ALL TEAM MEMBERS, or severe penalties will be incurred. Please refer to the section on plagiarism later in this document.

All team members will contribute equally to a written report and all will partipate in a video explaining their results and their contribution.

The folowing table is an *estimate* of the in-term assessment timing. Minor variations may occur.

Task	Date Avail.	Date Due	Form of Submission	Weighting
Short test	Fri Week 2	Fri Week 3	Written	0%
Project 1	Fri Week 3	Fri Week 6	Written and Video	20%
Project 2	Mon Week 7	Fri Week 9	Written and Video	20%

Late assessments will not be accepted.

6.2 Examination

Duration: Two hours.

Rationale: The final examination will assess student mastery of the material covered in the lectures.

Weighting: 60% of your final mark. Further details about the final examination will be available in class closer to the time. You will be required to submit a written exam and participate in a video explaining your exam answers.

7 Additional resources and support

7.0.1 Tutorial Exercises

A set of tutorial exercises will be given out. These problems are for YOU to do to enhance mastery of the course. SOME of the problems will be done in tutorials, but you will learn a lot more if you try to do them before the tutorial.

7.0.2 Lecture notes

Lecture notes will be displayed via a document camera through Blackboard Collaborate in moodle. In the event that face-to-face lectures become possible, lectures will additionally be simultaneously streamed and recorded.

7.0.3 Textbooks

There is no set text for this course. The following books may be useful as additional references and contain material well beyond the scope of this course:

- Robert Devaney, “An Introduction to Chaotic Dynamical Systems”
- Clark Robinson, “Dynamical Systems: Stability, Symbolic Dynamics, and Chaos”
- Michael Brin and Garrett Stuck, “Introduction to Dynamical Systems”
- James Meiss, “Differential Dynamical Systems”
- Thomas Schreiber and Holger Kantz, “Nonlinear Time Series Analysis”
- Anatole Katok and Boris Hasselblatt, “Introduction to the Modern Theory of Dynamical Systems”

7.0.4 Moodle

All course materials will be available on moodle. You should check regularly for new materials or updates.

8 Course Evaluation and Development

The School of Mathematics and Statistics evaluates each course each time it is run. We carefully consider the student responses and their implications for course development. It is common practice to discuss informally with students how the course and their mastery of it are progressing.

9 Administrative matters

9.0.1 School Rules and Regulations

Students must read and understand the School of Mathematics and Statistics Policies as contained in the ‘Important Information for Students’ document. This can be found on the web at `http://www.maths.unsw.edu.au/currentstudents/assessment-policies`

9.0.2 Plagiarism and academic honesty

Plagiarism is the presentation of the thoughts or work of another as one’s own. Issues you must be aware of regarding plagiarism and the university’s policies on academic honesty and plagiarism can be found at `https://student.unsw.edu.au/plagiarism`