Geometry & Groups

Stephan Tillmann

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Course description

According to Felix Klein's influential Erlanger program of 1872, geometry is the study of properties of a space, which are invariant under a group of transformations. In Klein's framework, the familiar Euclidean geometry consist of n-dimensional Euclidean space and its group of isometries. In general, a geometry is a pair (G,X), where X is a (sufficiently nice) space and G is a (sufficiently nice) group acting on the space. Geometric properties are precisely those that are preserved by the group. A geometry in Klein's sense may not allow the concepts of distance or angle; an example of this is affine geometry.

The study of geometry in Klein's framework motivates key ideas in different areas of mathematics, such as group theory, algebraic topology, differential geometry and representation theory. The seminal work of Bill Thurston has provided even more links between seemingly disparate fields of mathematics.

The aim of this course is to give students an introduction to geometry in the sense of Klein and Thurston, and to provide them with working knowledge of a variety of concepts and tools that are applicable in different fields of mathematics, as well as to open avenues for further study. A great emphasis will be placed on the detailed study of key examples.

About these notes

These notes accompany the course "Geometry & Groups" to be given by the author at the AMSI summer school at UNSW in 2012. This overview provides a guide to the 19 lectures with regards to contents, exercises and associated reading. However, this plan is likely to evolve during the course of the summer school.

The complete set of notes is expected to consist of six parts:

§0 Overview and bibiliography

§1 Möbius transformations and inversions

§2 Metric spaces

§3 Groups and group actions

§4 Some algebraic topology

§5 Geometric manifolds