The University of New South Wales
School of Mathematics and Statistics

Mathematics Drop–in Centre

BASIC ALGEBRA

The aims of algebra (up to first year) are solving equations and simplifying complicated expressions. You should know the basics from school, but a list of "do's and don'ts" may be useful.

Do solve equations by treating them like a balanced pair of scales. You can perform any legitimate operation you like on one side of the equation, as long as you perform the same operation on the other side at the same time. For example, to solve $2x+3 = 4x-5$ you could subtract $4x$ from both sides to get $-2x+3 = -5$, then subtract 3 from both sides to get $-2x = -8$, then divide both sides by -2 to arrive at the solution $x = 4$.

Don't use fake "rules" suggested by false analogies. Addition, subtraction, multiplication and division are all different and they obey different rules. For example, just because

$$x(y+z) = xy + xz$$

does not mean that

$$\frac{x}{y+z} = \frac{x}{y} + \frac{x}{z} \ .$$

You surely know that you must never divide by zero. But even more than this, **don't** ever divide by anything, or cancel anything, that *could possibly* be zero. For example, suppose you are asked to solve the equation

$$x(2x+3) = x(4x-5) \ .$$

You might begin by dividing both sides by x to obtain $2x+3 = 4x-5$, and then continue as above. However, this solution would be incomplete, because you have divided by x, which *might* be zero – after all, x is an unknown and you don't know what it is. A correct solution would be to first consider the possibility $x = 0$ and confirm that this is a solution; then consider the possibility that $x \neq 0$, in which case the above reasoning is correct. Therefore the equation has solution $x = 0$ or $x = 4$. An alternative would be to collect terms on one side of the equation and solve a quadratic.

If you take the square root of each side of an equation, **do** be sure to allow for the fact that a non–negative number will have two square roots. For example, the first step in solving

$$(2x+3)^2 = (4x-5)^2 \qquad (*)$$

is **not** $2x+3 = 4x-5$ but

$$2x+3 = \pm(4x-5) \ .$$

This then gives

$$2x+3 = 4x-5 \quad \text{or} \quad 2x+3 = -(4x-5)$$

and we find the solution $x = 4$ or $x = \frac{1}{3}$ (first part: as above, second: try it for yourself).

Simplifying an expression is a bit different from solving an equation. There are various methods you can use, and you will need to do lots of practice! Here is one example.

$$\begin{aligned}
\frac{x}{x-y} - \frac{y}{x+y} &= \frac{x(x+y) - y(x-y)}{(x-y)(x+y)} && \text{(common denominator)} \\
&= \frac{x^2 + xy - yx + y^2}{x^2 + xy - yx - y^2} && \text{(expand)} \\
&= \frac{x^2 + y^2}{x^2 - y^2} && \text{(cancel } xy \text{ terms)}
\end{aligned}$$

as long as $x \neq \pm y$.

EXERCISES.

Please try to complete the following exercises. Remember that you **cannot** expect to understand mathematics without doing lots of practice! Please do not look at the answers before trying the questions. If you get a question wrong you should go through your working carefully, find the mistake and fix it. If there is a mistake which you cannot find, or a question which you cannot even start, please consult your tutor or the Mathematics Drop–in Centre.

1. Solve the following equations:
   (a) $3x + 7 = 5x - 9$;
   (b) $5(x - 2) = 9(x + 4)$;
   (c) $(x - 2)(4x + 1) = (7x - 1)(x - 2)$;
   (d) $(2x - 1)(x - 5) = (2x - 1)(x + 7)$;
   (e) $x = \dfrac{3}{x}$;
   (f) $(x - 1)^2 = (3x + 1)^2$;
   (g) $(3x + 2)^2 = (4x - 7)^2$.
2. Decide whether the following algebraic "rules" are always true; or true with a small number of exceptions (give details); or usually false.
   (a) $xz - yz = (x - y)z$; (b) $(x + y)^{1/2} = x^{1/2} + y^{1/2}$;
   (c) $\dfrac{x}{z} + \dfrac{y}{z} = \dfrac{x + y}{z}$.
3. Simplify the following expressions. **Note** that in some cases no particular simplification is possible – it is important to recognise this so that you don't waste time on futile problems!
   (a) $x(x - y + z) + y(y - z + x) + z(z - x + y)$;
   (b) $\dfrac{4x}{2xy - y^2} - \dfrac{y}{2x^2 - xy}$; (c) $\sqrt{x^2 - y^2}$.
4. Expand the expression $(2x + 3)^2 - (4x - 5)^2$, and use your answer to give a different method of solving $(*)$.

ANSWERS.

1. (a) $x = 8$;
   (b) $x = -\frac{23}{2}$;
   (c) $x = 2$ or $x = \frac{2}{3}$;
   (d) $x = \frac{1}{2}$; note that if $2x - 1 \neq 0$ we get $x - 5 = x + 7$, which is impossible, so there are no further solutions;
   (e) $x = \sqrt{3}$ or $x = -\sqrt{3}$; observe that we should begin by noting that x cannot be zero, but as $x = 0$ does not appear in the solution anyway, this is no problem;
   (f) $x = -1$ or $x = 0$;
   (g) $x = 9$ or $x = \frac{5}{7}$.
2. (a) true;
   (b) false;
   (c) true as long as $z \neq 0$.
3. (a) $x^2 + y^2 + z^2$;
   (b) $\dfrac{2x + y}{xy}$, or $\dfrac{1}{x} + \dfrac{2}{y}$,
   provided that $x \neq 0$ and $y \neq 0$ and $y \neq 2x$;
   (c) no simplification – **not** $x - y$.
4. $-12x^2 + 52x - 16$; set this equal to zero and then solve the quadratic.