The University of New South Wales

School of Mathematics and Statistics

Mathematics Drop–in Centre

BASIC DIFFERENTIATION

You need to know securely the derivatives of simple functions. Some of those given below can be calculated from others by using differentiation rules; however, you don't want to do this for functions you will be using frequently, and so we recommend that you memorise all of the following.

function	derivative
constant	0
x^n	nx^{n-1}
e^x	e^x
$\cos x$	$-\sin x$
$\sin x$	$\cos x$
$\tan x$	$\sec^2 x$
$\ln x$	$\dfrac{1}{x}$
$\dfrac{1}{x}$	$-\dfrac{1}{x^2}$
$\sqrt{x}$	$\dfrac{1}{2\sqrt{x}}$

Observe that the second last entry in the above table is a consequence of the second. We have

$$\frac{1}{x} = x^{-1} \quad \text{and so} \quad \frac{d}{dx}\left(\frac{1}{x}\right) = \frac{d}{dx}(x^{-1}) = (-1)x^{-2} = -\frac{1}{x^2} \ .$$

We have, however, given this its own place in the table, and have suggested that you memorise it, because many students make the mistake of saying that the derivative of $1/x$ is $\ln x$, which is the wrong way round. You can find the derivatives of reciprocal powers of x in a very similar way, for example,

$$\frac{d}{dx}\left(\frac{1}{x^8}\right) = \frac{d}{dx}(x^{-8}) = (-8)x^{-9} = -\frac{8}{x^9} \ .$$

The last entry in the table can also be found from the second,

$$\sqrt{x} = x^{1/2} \quad \text{and so} \quad \frac{d}{dx}(\sqrt{x}) = \frac{d}{dx}(x^{1/2}) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}} \ .$$

Other roots can be treated in the same way.

The **second derivative** of a function means the derivative of the derivative. For example, the derivative of x^7 is $7x^6$, and so the second derivative of x^7 is

$$\frac{d^2}{dx^2}(x^7) = \frac{d}{dx}(7x^6) = 7(6x^5) = 42x^5 \ .$$

If you differentiate again, you get the **third derivative**, and so on.

Notation. Please use notation accurately: $\frac{d}{dx}$ means "the derivative of", and $\frac{dy}{dx}$ means "the derivative of y". So "the derivative of x^5" is written $\frac{d}{dx}(x^5)$. Please do not write "$\frac{dy}{dx}(x^5)$", it is nonsense!!

EXERCISES.

Please try to complete the following exercises. Remember that you **cannot** expect to understand mathematics without doing lots of practice! Please do not look at the answers before trying the questions. If you get a question wrong you should go through your working carefully, find the mistake and fix it. If there is a mistake which you cannot find, or a question which you cannot even start, please consult your tutor or the Mathematics Drop–in Centre.

1. Write out the table of basic derivatives from memory.
2. You should be familiar with the "dash" notation for derivatives. For example, the second entry in the table can be stated as "if $f(x) = x^n$ then $f'(x) = nx^{n-1}$". Write out the whole table in this format.
3. Write down the derivatives of the following functions:

$$x^6\ , \quad x^{1/6}\ , \quad \frac{1}{x^6}\ , \quad \tan x\ , \quad \frac{1}{x}\ , \quad \ln x\ .$$

4. Find the derivatives of $\sqrt[3]{x}$, $\sqrt[4]{x^5}$, $x^{3\cdot 14}$, $x^{-3\cdot 14}$, $\cos x$.
5. (a) Find the second and third derivatives of $\ln x$.
   (b) Find the fourth derivative of $\sin x$.
   (c) Find the 99th derivative of e^x.
6. You need to be equally comfortable with differentiation if the variable is something other than x. For example, to find the derivative with respect to t of t^3 we write

$$\frac{d}{dt}(t^3) = 3t^2\ .$$

   Find, and write as an equation following the above example,
   (a) the derivative with respect to t of e^t;
   (b) the derivative with respect to θ of $\cos\theta$;
   (c) the *second derivative* with respect to z of z^4.

ANSWERS.

3. $6x^5$, $\frac{1}{6}x^{-5/6}$, $-\dfrac{6}{x^7}$, $\sec^2 x$, $-\dfrac{1}{x^2}$, $\dfrac{1}{x}$.
4. $\dfrac{1}{3}x^{-2/3}$, $\dfrac{5}{4}x^{1/4}$, $3\cdot 14x^{2\cdot 14}$, $-3\cdot 14x^{-4\cdot 14}$, $-\sin x$.
5. (a) $-\dfrac{1}{x^2}$, $\dfrac{2}{x^3}$.
   (b) $\sin x$.
   (c) e^x.
6. (a) $\dfrac{d}{dt}(e^t) = e^t$;
   (b) $\dfrac{d}{d\theta}(\cos\theta) = -\sin\theta$;
   (c) $\dfrac{d^2}{dz^2}(z^4) = 12z^2$.