2021 Honours Projects in Applied Mathematics

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This booklet contains descriptions of thesis projects offered for Honours year students in Applied Mathematics. Honours candidates are strongly encouraged to contact their preferred supervisor as early as possible to discuss potential projects and to make sure they have any requisite background knowledge. More information about the Honours year is available by emailing the Applied Mathematics honours coordinator or at

https://www.maths.unsw.edu.au/currentstudents/honours-mathematics-and-statistics.

Contents

1 Biomathematics

1.1 Christopher Angstmann

1.1.1 Models of cell division in bacteria and archaea

This project will look at the remarkable Turing instabilities in sets of reaction-diffusion equations that allow bacteria and archaea to divide in half. Bacteria and archaea are two of the three domains of life. Both bacteria and archaea are prokaryotic cells without any membrane bound organelles. In these cells, the motion of proteins and other biomolecules is typically purely diffusive, without the motor driven directed transport seen in Eukarya. As such reaction-diffusion models serve to describe the dynamics of these proteins, including the sets of proteins responsible for finding the midpoint of the cell for division. This project will construct models of cell division, based on reaction-diffusion equations, in two and three dimensions and compare these with experimental results.

1.2 Adelle Coster

Email A.Coster@unsw.edu.au if you are interested in these or other biomathematical projects. Please include some information about your mathematical background and interests.

1.2.1 The insulin signalling pathway in adipocytes: a mathematical investigation.

The insulin signalling pathway in adipocytes (fat cells) is the main controller of the uptake of glucose in the cells. Understanding of this system is vital in the investigation of diabetes, which is a deficiency in this control system. Projects in this area include the development a mathematical description of the movement of the vesicles (small membrane spheres) containing glucose transporter proteins, and also the diffusion of the proteins if the vesicle fuses with the cell surface, and also the analysis of the biochemical signalling pathway from the insulin receptor. These will involve analysis of differential equations and possibly some computational simulations of the system. Experimental data for comparison with the models will be available

1.2.2 Do glucose transporters queue to get to the cell surface?

Cells transport glucose into their interior via protein channels. In adipocytes (fat cells) glucose uptake is regulated by insulin, as a dynamic balance between exocytosis (outward transport) and endocytosis (inward transport) of the proteins. The proteins are, however, packeted into small vesicles (spheres of membrane) when transported. One characterisation of the observed transport behaviour utilises queueing theory. This idea stems from the presence of the microtubules that cross the cytoplasm of the cells. Microtubules are implicated in the sorting of different endocytic vesicular contents and have well characterised molecular motors which control the movement of vesicles down their lengths. Vesicles carrying the proteins have been observed to be transported along microtubules. Could these act as a scaffold for the vesicles, which would then form queues waiting for exocytosis? This project will model the recycling system as a closed Markovian queueing network and explore what characteristics of the network may be responsible for the transitory behaviour when insulin is applied, initiating protein transport. Experimental data for comparison will be available and the investigation will have both theoretical and computational aspects.

1.2.3 Modelling Myelinated Nerve Function.

Pacemaking in cardiac and neuronal cells is primarily controlled by the interaction between different voltage gated ion channels. An existing mathematical model of the human motor axon utilises coupled differential equations to describe the electrical activity within the nerve. This model will be explored to interpret the measured responses to extended hyperpolarisation and the contribution of different ion currents to the transmission of signals. It will also investigate the responses of sensory neurons to strong longlasting hyperpolarisation and contrast with motor neurons. Extensions to the model will then be made to incorporate additional inhomegeneities within the neuron structure. This project will include analysis and development of coupled differential equations, numerical simulations and the optimisation of the model using data from the experimental literature.

1.3 Gary Froyland

1.3.1 Lagrangian coherent structures in haemodynamics

Haemodynamics (the dynamics of blood flow) is believed to be a crucial factor in aneurysm formation, evolution, and eventual rupture. Turbulent motion near the artery wall can weaken already damaged arteries, as can oscillations between turbulent and laminar flow. Simulations of 3D blood flow is either derived by (i) computational fluid dynamics (CFD) from patient-specific mathematical models obtained from angiographic images or (ii) laser scanning of real flow through a patient-specific physical plastic/gel cast. In this project, joint with A/Prof. Tracie Barber (UNSW Mech. and Manufact. Engineering), you will apply the latest mathematical techniques for flow analysis, based on dynamical systems and elliptic PDEs to separate and track regions of turbulent and regular blood flow.A/Prof. Barber will provide the realistic flow data from her laboratory, from both CFD simulations and physical casts. There is also the opportunity to further develop mathematical theory to solve problems specific to haemodynamics.

1.4 Anna Cai

Please contact Dr Cai directly (a.cai@unsw.edu.au) for potential honours projects.

2 Computational Mathematics

2.1 Zdravko Botev

2.1.1 High-Dimensional numerical integration with applications in Bayesian statistics.

One of the simplest mathematical models for measurements $\boldsymbol{y} = (y_1, \ldots, y_m)^\top$ requires that we estimate or approximate accurately a high dimensional integral of the form

$$\iint \exp\left(-\frac{\|\boldsymbol{y} - \mathrm{X}\boldsymbol{\beta}\|^2}{2\sigma^2} - m\ln\sigma - \sum_i \lambda_i |\beta_i/\sigma|^\alpha\right) \mathrm{d}\boldsymbol{\beta}\mathrm{d}\sigma,$$

where $\alpha \in (0, 1]$ and $\lambda_i > 0$ are regularization parameters and X is a fixed matrix. In this project you will explore a nested integration method for computing integrals like the above one. The project will not only explore the estimation of high-dimensional integrals, but also their relation to random variable simulation in Bayesian statistics.

2.1.2 Monte Carlo splitting method for integrals with quasi-monotone integrands.

Consider the numerical approximation of the small probability $\ell = \mathbb{P}(S(\boldsymbol{X}) \geq \gamma)$, where $\boldsymbol{X}$ is drawn from a d-dimensional probability density function f, the threshold γ is large enough to make ℓ very small, and S is a quasi-monotone function (e.g., $x_i \leq y_i,\ i = 1, \ldots, d$ implies $S(\boldsymbol{x}) \leq S(\boldsymbol{y})$). We thus need to compute the high-dimensional integral

$$\ell = \int_{S(\boldsymbol{x})\geq\gamma} f(\boldsymbol{x})\mathrm{d}\boldsymbol{x}$$

Such high-dimensional integrals with non-smooth integrands frequently arise in portfolio credit risk assessment, where estimating ℓ is of interest, because it is the probability of bankruptcy or default.

In this project you will explore the properties of a Monte Carlo splitting algorithm for computing ℓ. Monte Carlo splitting is a method that splits paths of a random process evolving over time. This project may involve collaboration with researchers in Rennes in France and Montreal in Canada.

2.2 Gary Froyland

2.2.1 Dynamical kernel methods for big data

This project will study data-driven kernel methods to analyse high-dimensional time series. The building blocks will be transfer operators and dynamic Laplace operators, which extract dominant modes from the data. This project will investigate the construction of these operators from high-dimensional data using dynamical kernel methods. A possible application is to analysing global scalar fields obtained from satellite imagery such as sea-surface temperature to extract climate oscillations such as the El Nino Southern Oscillation and the Madden-Julian Oscillation. This project will use ideas from dynamical systems, functional analysis, and Riemannian geometry.

2.3 Frances Kuo

Quasi-Monte Carlo methods, or QMC methods in short, are numerical methods for high dimensional integration and related problems. The prefix "quasi" indicates that these methods rely on cleverly designed pointsets or sequences in high dimensions, as opposed to the regular Monte Carlo method which is based on sequences of pseudorandom numbers. Families of QMC methods have been around since the late 1950s, but for a long time it was believed that they would not be effective in high dimensions due to an exponential growth of the theoretical error bound in high dimensions. However, researchers have demonstrated in the mid 1990s that QMC methods can be very effective even in hundreds of dimensions for some finance problem from Wall Street. Since then there has been a huge surge of research developments in QMC, and there is a strong group based at UNSW Sydney. We now know how to construct good QMC methods efficiently in thousands of dimensions, with fast error convergence independently of dimension!

A potential honours project would be on the theoretical development and/or practical application of QMC methods.

2.4 Quoc Thong Le Gia

2.4.1 Approximate cloaking simulation

Optical cloaking has been and continues to be a fascinating subject. Invisibility has been a subject of human fascination for millennia, from the Greek legend of Perseus versus Medusa to the more recent *The Invisible Man*. Since 2005 there has been a wave of serious theoretical proposals in the physics literature, and a widely reported experiment by Schurig et al., for cloaking devices – structures that would not only render an object invisible but also undetectable to electromagnetic waves. The mathematical foundations of optical cloaking are described in an excellent article by Greenleaf et.al. and Vogelius et. al.

The transformation optics approach to cloaking uses a singular change of coordinates which blows up a point to the region being cloaked, is singular and hence it is difficult to analyse theoretically. Hence a rigorous numerical simulation will shed the light on the problem significantly.

In the project, you will investigate the numerical simulation of an approximate cloak over a circular domain with a fixed wave number.

2.4.2 Analysis of changing data and applications

A unified framework for performing classification, regression, manifold learning, and related tasks from data analysis consists of approximating functions of the form $f : X \subset \mathbb{R}^d \mapsto \mathbb{R}^q$ (for some fixed $d, q \in \mathbb{N}$), given a finite amount of information about the function. We will refer to problems of changing data as those where the functions of interest are known on dissimilar domains.

As an application of changing data problem, we will look at a sequence of MRI brain scans of patients with Alzheimer's disease (AD) at regular intervals over several years. It is known that the cortical thickness will change along with the disease's progression. A changing-data problem of clinical significance is quantifying this change given a sequence of MRI brain scans for a particular patient, and using this information to discern between healthy and AD patients.

We also look at the mathematical tools which could be used to solve the changing data problem, namely polynomial frame operators, scattered data approximation methods, etc.

2.5 Bill McLean

2.5.1 Hierarchical Matrices.

In 1999, Wolfgang Hackbusch introduced the notion of a hierarchical matrix or H-matrix. The idea grew out of a computational algorithm for solving integral equations, known as panel clustering. For an $N \times N$, dense matrix, computing a matrix-vector product in the obvious way costs N^2 operations. However, for a wide class of matrices arising as discretizations of integral and other linear operators, an H-matrix representation uses only $O(N)$ storage and allows the computation of a matrix-vector product using only $O(N)$ operations, accurate to within the order of the discretization error. It is even possible to develop a whole fast algebra of matrices, with many interesting applications for large-scale numerical simulations. A project in this area would involve a mix of theory and computation, depending on the background of the student.

2.5.2 Approximating the fractional powers of an elliptic differential operator

A number of interesting applications lead to fractional order partial differential equations, and in particular to equations involving a fractional power of an elliptic operator. Computational schemes for solving such equations rely on efficient and accurate numerical methods to approximate such an operator. The aim of the project is to compare a few such methods, setting out the pros and cons of each approach.

2.5.3 Localization of eigenfunctions

If a quantum system has a regular potential, then a typical eigenstate is a smooth, oscillatory function with global support. However, when the potential energy is sufficiently disordered, a phenomenon called Anderson localization can occur, in which an eigenstate is a spike, that is, large in a small region of the spatial domain with rapid decay away from that region. The project will involve a mix of studying the known theoretical results on this effect, and performing direct numerical simulations.

2.6 Thanh Tran

2.6.1 The role of the Landau-Lifshitz-Gilbert equation in the theory of novel magnetic memories.

One of the hallmarks of modern society is the increasing demand for the large data storage which can be rapidly and efficiently accessed. The most important devices for information storage are magnetic memories which are used in, for example, mobile phones, credit cards, televisions, and computer hard drives.

Submicron-sized ferromagnetic elements are the main building blocks of data storage devices. They preserve magnetic orientation for a long time, allowing bits of information to be encoded as directions of the magnetisation vector. The stored information can be modified by an external magnetic field.

A well-known model for magnetisation is the Landau-Lifshitz-Gilbert equation. The equation possesses complex mathematical properties such as nonconvex side-constraints, strongly nonlinear terms and the appearance of singularities. These properties demand sophisticated numerical approximations.

In this project you will learn different numerical methods to solve the equation. Depending on your needs and interests, there are open problems to cut your teeth on.

2.6.2 Problems in random domains.

In many industries (e.g., in aerospace engineering) random discrepancies between the ideal geometries conceived in the design phase and their actual realisation may lead to considerable variations in expected outcomes.

The effect of randomness in domains is even more dramatic in manufacturing of nano-devices (e.g. data storage devices governed by the Landau-Lifshitz-Gilbert equation). Indeed, under certain resolution, surfaces of these devices become rough, and a minor discrepancy results in relatively large adverse effects.

In this project you will learn how shape calculus can be used to deal with problems on random domains.

2.6.3 Boundary element methods.

Boundary element methods have long been used in engineering to solve boundary value problems. These problems are formulated from many physical phenomena, ranging from mechanical engineering (e.g., in car design) to petroleum engineering (e.g., for simulation of fractured reservoirs). In this project, you will first learn basic concepts of boundary element methods, how to implement and analyse efficiency and accuracy of the methods. Then, depending on your needs and interests, you will use the methods to solve practical problems in engineering or geodesy. Problems in geodesy will involve programming with data collected by a NASA satellite, which may contain up to almost 30 million points.

3 Fluid dynamics, oceanography and meteorology

3.1 Gary Froyland

3.1.1 Lagrangian Coherent Structures in Ocean and Atmosphere Models

The ocean and atmosphere display complex nonlinear behaviour, whose underlying evolution rules change over time due to external and internal influences. Mixing processes of in the atmosphere and the ocean are also complex, but carry important geometric transport information. Using the latest models or observational data, and methods from dynamical systems, and elliptic PDEs, this project will identify and track over time those geometric structures that mix least. Known examples of such structures are eddies and gyres in the ocean, and vortices in the atmosphere, however, there are likely many undiscovered coherent pathways in these geophysical flows. There is also the possibility for the project to further develop mathematical theory and/or algorithms to treat one or more specific challenges arising in these application areas. This could a joint project with Mark Holzer or Shane Keating.There is a possibility to undertake a joint project with Mark Holzer or Shane Keating.

3.2 Mark Holzer

3.2.1 New constraints on large-scale tropospheric transport from global trace-gas measurements.

Use forward and/or inverse modeling of trace gases with a range of chemical lifetimes to extract the transport paths and timescales with which the highly turbulent atmosphere distributes pollutants and greenhouse gases.

3.2.2 Ocean biogeochemistry.

Couple models of biogeochemical cycles to ocean circulation models and to each other to quantify the key nutrient cycles and elemental ratios in the ocean. Learn about the oceans biological pump and atmospheric CO2 regulation.

3.2.3 Construction of matrix models for geophysical flows.

Develop numerically efficient models of geophysical flows within Matlab and investigate their transport properties using Green-function diagnostics.

3.2.4 Turbulence modeling.

Develop stochastic turbulence models and investigate their transport using Green-function tracers.

3.3 Shane Keating

3.3.1 Simulating fractal curves in turbulent fluid flows

A patch of dye immersed in a turbulent flow tends to be stretched and deformed into strikingly convoluted, fractal-like patterns, like cream stirred into coffee. This fractal structure provides a fingerprint of the underlying flow and is intimately linked with the processes of stirring and mixing in turbulence. In this project, we will investigate the theoretical connection between the fractal geometry of material fields (like dye) and diffusion in turbulent flows. We will also investigate novel stochastic methods for generating fractal Gaussian fields with the goal of representing unresolved mixing in numerical simulations of turbulence. This project is co-supervised by Zdravko Botev. Some knowledge of Matlab will be required.

3.3.2 Ocean current velocimetry from ultra-high resolution satellite imagery

Particle image velocimetry (PIV) is a widely used technique for measuring flow velocities by tracking features in sequential images. In this project, we will test the feasibility of using PIV to estimate ocean currents at the surface of the ocean from satellite images of sea-surface temperature. Knowledge of satellite remote sensing and ocean dynamics is not required, but strong computational ability in Matlab or Python is a must.

3.3.3 Fluid transport by vortex ring entrainment

Vortices — rotating bodies of fluid that remain coherent for long periods — are frequently observed in the atmosphere, ocean, and in laboratory experiments. Observations and simulations of vortices indicate that they are important for transporting properties such as heat, biological material, or pollutants over large distances. While some fluid is transported by the core of the vortex, there is also transport due to ambient fluid that is captured or "entrained" within the outer ring and then travels with the vortex as it propagates. In this project, we will examine transport by entrainment of fluid in the vortex ring, or of multiple vortex rings. Experience with Python is required.

3.3.4 Fluid dynamics of cycling peloton formation

In competitive cycling, a "peloton" is a group of riders that travels together in formation in order to reduce drag and save energy. The shape of the peloton will change depending on headwind and sidewind and the strategy of individual riders or teams of riders. In this project, we will study the fluid dynamics of cycling pelotons and investigate how collective behavior of cyclists can lead to peloton formation under different scenarios. Python programming experience is essential.

3.4 Trevor McDougall

3.4.1 Ocean mixing and the absolute velocity in the ocean

For much of the history of physical oceanography a challenging quest has been to deduce the mean velocity of the ocean from shipboard measurements of density as a function of depth, longitude and latitude. These measurements constrain the vertical variations of the horizontal velocity, but do not constrain a depth-independent offset, and solving for this reference value of velocity has been a major focus of blue water oceanography during the whole of the 20th century. Recent theoretical work has yielded a closed expression for the absolute velocity which includes the depth-independent reference velocity. This closed expression reveals the role of ocean mixing processes in driving the mean circulation, and it also demonstrates the importance of the geometry of surfaces of constant temperature and salinity. This project will analyze numerical ocean model output to explore the dominant balances which allow the ocean to move, concentrating in the Southern Ocean. There are several levels of sophistication that can be applied to this closed expression for the absolute velocity, and the project will aim to pick the lowest-hanging fruit first, concentrating on finding spatial regions where the inverse technique is most likely to work, using a recently deduced predictive measure of the likely signal-to-noise of the inversion. The overall aim of doing inverse studies such as this is to estimate the strength of mixing processes in large regions of the ocean. These mixing intensities are needed as inputs into ocean and climate models. This project is co-supervised by Prof. Trevor McDougall and Dr Sjoerd Groeskamp.

3.4.2 Forming the integrating factor for Neutral Density in the ocean

The concept of a neutral tangent plane in the ocean is well established, and it describes the local plane in which the very strong lateral mixing of mesoscale eddies occurs. The diffusivity of a passive tracer in this plane is ten million times greater than in the direction normal to this plane. Because of this large difference in the amount of mixing in these different directions, it is important for oceanography and for climate science to be able to accurately calculate the neutral direction in space. While the local direction is unambiguous, it is difficult to find a globally extensive surface. We do have a differential equation for the integrating factor needed to form Neutral Density; that is, for the integrating factor that when multiplied by a known density gradient will give the gradient of Neutral Density. This project will explore ways of evaluating the integrating factor in the ocean, as a necessary prelude to calculating Neutral Density in space. This project is co-supervised by Prof Trevor McDougall, Dr Paul Barker and Dr Casimir de Lavergne.

3.5 Amandine Schaeffer

3.5.1 Dynamics of a marine heatwave: what happens below the surface?

Extreme temperatures in the ocean are getting more frequent and intense, impacting marine ecosystems and industries. However the subsurface signature of these marine heatwaves is still largely unknown, in particular in shallow coastal areas where most of the ecological damages occur.

In addition to sustained observations, the Australian Integrated Marine Observing System (IMOS) now aims at sampling the coastal ocean during marine heatwaves with real-time deployments of ocean gliders. Gliders are automated underwater vehicles which measure the water properties between the ocean floor and the surface for a few weeks. Two of such deployments were successfully finalised, sampling the eastern shelf of Tasmania during the latest marine heatwave event in the Tasman Sea in summer / spring 2019.

The project aims at understanding the extent and characteristics of marine heatwaves using glider measurements and complementary satellite and moored observations. Key questions include the temporal evolution, from the onset to the decline of the extreme event, and the influence of the local oceanography such as currents and wind-driven processes on the persistence and variability of these anomalous temperatures. The student will use programming language to analyse this unique dataset and compute the heat budget equations.

Basic knowledge of oceanography and experience in Matlab or Python are required. The project will be based at UNSW Sydney, co-supervised by Amandine Schaeffer (UNSW), Jessica Benthuysen (AIMS) and Neil Holbrook (UTAS).

3.6 Chris Tisdell

3.6.1 Exploring the theory of Navier-Stokes equations and their applications to fluid flow

Navier-Stokes equations are of immense theoretical and physical interest. These partial differential equations have been used to better understand the weather, ocean currents, water flow in a pipe and air flow around a wing. However, the theory of the equations has not yet been fully formed. For example, it has not yet been proven whether solutions always exist in three dimensions and, if they do exist, whether they are smooth - i.e. they are infinitely differentiable at all points in the domain. The Clay Mathematics Institute has identified this as one of the seven most important open problems in mathematics and has offered a US$1 million prize for a solution or a counterexample.

In this project we will examine existence and smoothness of solutions to problems derived from the Navier-Stokes equations that arise in laminar fluid flow in porous tubes and channels. Channel flows - liquid flows confined within a closed conduit with no free surfaces - are everywhere. In plants and animals they serve as the basic ingredient of vascular systems, distributing energy to where it is needed and allowing distal parts of the organism to communicate. In engineering, one of the major functions of channels is to transport liquids or gases from sites of production to the consumer or industry.

Such a project will involve the nonlinear analysis of boundary value problems and some numerical approximations.

3.7 Jan Zika

3.7.1 Distilling the oceans role in climate using thermodynamic diagrams

Understanding how much and to what depth heat will be pumped into the ocean is critical to predict future surface temperature and sea-level rise.

This study will investigate vertical heat transport in the ocean using novel thermodynamic diagrams. Using such diagrams, which have origins in classical thermodynamics, one can relate the circulation to surface heating and cooling processes and mixing.

Solutions for such circulations are tractable both from analytical, simple numerical and observational points of view. The student will consider both idealised cases and make use of the most recent observations. These will be combined with constraints based on theories of ocean mixing and energetics to generate estimates of the deep overturning circulation and its role in transient climate change.

3.7.2 Linking the seasonal cycle of ocean water masses to transient climate change

In boreal winter the North Atlantic and Pacific Oceans become cold, dense and turbulent. Oxygen, carbon and other substances are drawn out of the atmosphere and ventilated into the deep ocean. In boreal summer, as the surface layers in the north warm, cooling and ventilation begins in the southern hemisphere in earnest.

The process of seasonal ventilation dictates the oceanÕs role in climate - both present and future. Only in the last decade has a systematic understanding of seasonal ventilation become possible due to the presence of thousands of autonomous buoys (ARGO) and satellites measuring upper ocean temperature and salinity. Likewise never has the need to quantify it been more pressing.

This project will combine the latest observations to generate a quantitative picture of the formation, ventilation and destruction of cold dense water masses in both hemispheres. A key novelty of this project will be the use the water-mass transformation framework. Using this framework variability in water mass properties into that due to surface heating and cooling, evaporation and precipitation, mixing and energetic drivers such as wind forcing.

3.7.3 Asymmetry of the ocean's thermohaline circulation

The ocean is highly turbulent. Pathways of free-floating buoys are chaotic and circulation patterns are dominated by mesoscale eddies, the ocean's equivalent to atmospheric storms. The ocean is at the same time organized. Substances injected into the ocean follow broad and distinct routes near the sea surface from the Pacific to the Atlantic Ocean. As a result the North Pacific and North Atlantic Ocean's are in marked contrast. The Pacific is cold and fresh and the Atlantic is warm and salty. Known as the thermohaline circulation, this helps maintain

Europe's relatively mild climate.

This project will explore the link between the asymmetry in northern hemisphere climates, the thermohaline circulation and the atmospheric forcing which sets the eventual temperature and salinity of sea-water. The project will pivot on the hypothesis that, by accident of geography and the position of southern hemisphere winds, warm saline water preferentially flows into the Atlantic. Moreover these effects will dictate the stability of the thermohaline circulation and European climate over coming centuries.

4 Mathematics Education

4.1 Chris Tisdell

4.1.1 Digital resources in mathematics: What makes them effective for learning?

The past 10 years has seen an explosion in the creation of digital resources for learning mathematics via the web. This thesis will investigate best practice in the design, development and delivery of digital learning resources and will discover how students engage with them. This project is suitable for anyone who is passionate about quality in the teaching and learning of mathematics via digital platforms.

4.1.2 Mathematical learning communities

The first year university student experience plays an important formative role in shaping student attitudes and presents an opportunity to build a sense of community and enhance approaches to learning. This project will investigate the idea of a learning community for those first-year students taking mathematics courses as UNSW. The project will explore questions such as: What is a learning community in mathematics? Do they exist at UNSW? What makes an effective learning community in mathematics? What are the benefits of peer-to-peer learning? This project will be suitable for anyone who is interested in building better learning communities in mathematics at UNSW.

4.1.3 Peer to Peer Support for Mathematics Students

This project will investigate the concept of peer-to-peer learning for students in mathematics courses. Important research questions associated with this investigation include: What models are there for peer-to-peer learning? What are student and teacher attitudes towards peer-to-peer learning? And what works best?

4.1.4 Learning Mathematics on the Move via Mobile Devices

Smart phones, tablets and laptops have become central technological tools for learning in the past 10 years. This project will look at best practice for mobile learning (MLearning) for mathematics. What is current practice and where are we going? What works well and what can be improved?

5 Nonlinear Phenomena

5.1 Christopher Angstmann

5.1.1 Random walks and fractional calculus

There is a remarkable connection between a class of non-Markovian stochastic processes, typically cast as random walks, and fractional order calculus. This connection can be exploited in a wide variety of ways, from building better models involving fractional derivatives, to the creation of novel numerical methods. There are a variety of potential honours projects that could be given to students who are interested in learning more about fractional calculus.

5.2 Gary Froyland

5.2.1 Topics in dynamical systems, ergodic theory, and differential geometry

Ergodic theory is the study of the dynamics of ensembles of points, in contrast to topological dynamics, which focusses on the dynamics of single points. A number of theoretical Honours projects are available in dynamical systems, ergodic theory, and/or differential geometry, aiming at developing new mathematics to analyse the complex behaviour of nonlinear dynamical systems. Depending on your background, these projects may involve mathematics from Ergodic Theory, Functional Analysis, Measure Theory, Riemannian Geometry, Stochastic Processes, and Nonlinear and Random Dynamical Systems.

5.2.2 Transfer operator analysis with applications to fluid mixing.

A transfer operator is a linear operator that completely describes the evolution of probability densities of a nonlinear dynamical system. Transfer operators are therefore fundamental objects like discrete-time maps and continuous time flows, but operate on ensembles rather than single points. Spectral techniques using transfer operators have recently been shown to be particularly effective for analysing complex dynamics in a variety of theoretical and physical systems, and are an active research area internationally. This project will focus on developing powerful transfer operator techniques to extract important differential geometric and probabilistic dynamical structures from fluid-like models. If desired, application areas include the ocean (an incompressible fluid) and the atmosphere (a compressible fluid).

5.2.3 Extreme value statistics for chaotic systems.

Accurately estimating the probability of rare events is particularly challenging in models with long memory, such as systems with a high level of determinism and a low level of randomness. This project will develop mathematical theory and accurate, rigorous numerical schemes to handle such systems. The project will also apply these new methods to estimate the likelihood of rare events from real data. The project will use mathematics from probability and statistics, functional analysis, and connects to dynamical systems and ergodic theory.

5.3 Chris Goodrich

5.3.1 Positivity, Monotonicity, and Convexity Results for Discrete Fractional Operators

Description: Consider a function f defined on the set $\mathbb{N}_a := \{a, a+1, a+2, \dots\}$ for some $a \in \mathbb{R}$. The first-order difference of f at $t \in \mathbb{N}_a$ is then defined by $(\Delta f)(t) := f(t+1) - f(t)$. It is an obvious consequence of $(\Delta f)(t) \geq 0$ that f is increasing at t. Recently there has been much work in developing a discrete fractional calculus. Discrete fractional operators not only take into account the values of f at local time points (e.g., t and $t+1$ as above), but, in fact, take into account a weighted average of the values of f at ***all*** admissible time points. This means that the relationship, for example, between the sign of a fractional difference of f and the monotonicity (i.e., increasing or decreasing) behaviour of f is much more complicated than in the integer-order difference case. We will explore various conditions on fractional order differences that imply (or fail to imply) not only monotonicity-type results but also positivity and convexity results.

5.4 John Roberts

5.4.1 Algebraic dynamics.

The topic is broadly taken to be the intersection of algebra, number theory, and dynamical systems. This interdisciplinary area of research is cutting edge and exciting, and has important applications to, e.g., cryptography, random matrix theory, materials science and engineering.

5.4.2 Discrete integrable systems.

The study of integrable (partial) difference equations and integrable maps is presently a very active field of research. In the first instance, this is due to the increasingly numerous areas of physics in which such systems feature. The study of discrete integrable systems also has intrinsic mathematical appeal, broadly speaking to do with finding analogues of concepts or properties (e.g., the Painlevé property, Lax pairs, Hamiltonian structure) that exist in integrable systems with continuous time. Increasingly, I am interested in using algebraic geometry and ideal theory to understand discrete dynamical systems.

5.5 Wolfgang Schief

5.5.1 Topics in soliton theory.

Solitons constitute essentially localised nonlinear waves with remarkable novel interaction properties. The soliton represents one of the most intriguing of phenomena in modern physics and occurs in such diverse areas as nonlinear optics and relativity theory, plasma and solid state physics, as well as hydrodynamics. It has proven to have important technological applications in optical fibre communication systems and Josephson junction superconducting devices.

Nonlinear equations which describe solitonic phemonema ("soliton equations" or "integrable systems") are ubiquitous and of great mathematical interest. They are privileged in that they are amenable to a variety of solution generation techniques. Thus, in particular, they generically admit invariance under symmetry transformations known as *Bäcklund transformations* and have associated nonlinear superposition principles (*permutability theorems*) whereby analytic expressions descriptive of multi-soliton interaction may be constructed. Integrable systems appear in a variety of guises such as ordinary and partial differential equations, difference and differential-difference equations, cellular automata and convergence acceleration algorithms.

It is by now well established that there exist deep and far-reaching connections between integrable systems and classical differential geometry. For instance, the interaction properties of solitons observed in 1953 by Seeger, Donth and Kochendörfer in the context of Frenkel and Kontorovas dislocation theory, and later rediscovered by Zabusky and Kruskal (1965) in connection with the numerical treatment of the important Fermi-Pasta-Ulam problem, are encoded in the geometry of particular classes of surfaces governed by the sine-Gordon equation and Korteweg-de Vries (KdV) equation respectively. The geometric study of integrable systems has proven to be very profitable to both soliton theory and differential geometry.

Integrable systems play an important role in *discrete differential geometry*. The term "discrete differential geometry" reflects the interaction of differential geometry (of curves, surfaces or, in general, manifolds) and discrete geometry (of, for instance, polytopes and simplicial complexes). This relatively new and active research area is located between pure and applied mathematics and is concerned with a variety of problems in such disciplines as mathematics, physics, computer science and even architectural modelling. Specifically, theoretical and applied areas such as differential, discrete and algebraic geometry, variational calculus, approximation theory, computational geometry, computer graphics, geometric processing and the theory of elasticity should be mentioned.

Soliton theory constitutes a rich source of Honours topics which range from applied to pure. Specific topics will be tailored towards the preferences, skills and knowledge of any individual student.

5.6 Chris Tisdell

5.6.1 A deeper understanding of discrete and continuous systems through analysis on time scales.

Historically, two of the most important types of mathematical equations that have been used to mathematically describe various dynamic processes are: differential and integral equations; and difference and summation equations, which model phenomena, respectively: in continuous time; or in discrete time. Traditionally, researchers have used either differential and integral equations or difference and summation equations — but not a combination of the two areas — to describe dynamic models. However, it is now becoming apparent that certain phenomena do not involve solely continuous aspects or solely discrete aspects. Rather, they feature elements of both the continuous and the discrete. These types of hybrid processes are seen, for example, in population dynamics where non-overlapping generations occur. Furthermore, neither difference equations nor differential equations give a good description of most population growth. To effectively treat hybrid dynamical systems, a more modern and flexible mathematical framework is needed to accurately model continuous, discrete processes in a mutually consistent manner. An emerging area that has the potential to effectively manage the above situations is the field of dynamic equations on time scales. Created by Hilger in 1990, this new and compelling area of mathematics is more general and versatile than the traditional theories of differential and difference equations, and appears to be the way forward in the quest for accurate and flexible mathematical models. In fact, the field of dynamic equations on time scales contains and extends the classical theory of differential, difference, integral and summation equations as special cases. This project will perform an analysis of dynamic equations on time scales. It will uncover important qualitative and quantitative information about solutions; and the modelling possibilities. Students who undertake this project will be very well equipped to make contributions to this area of research.

5.6.2 Advanced Studies in differential equations.

Many problems in nonlinear differential equations can reduced to the study of the set of solutions of an equation of the form $f(x) = p$ in an appropriate space. This project will give the student an introduction to the applications of analysis to nonlinear differential equations. We will answer such questions as:

- When do these equations have solutions?
- What are the properties of the solution(s)?

- How can we approximate the solution(s)?

A student who completes this project will be well-prepared to make the transition to to research studies in related fields.

5.6.3 Advanced studies in nonlinear difference equations.

Difference equations are of huge importance in modelling discrete phenomena and their solutions can possess a richer structure than those of analogous differential equations. This project will involve an investigation of nonlinear difference equations and the properties of their solutions (existence, multiplicity, boundedness, etc). Students who complete this project will be very well-equipped to contribute to the research field.

6 Optimisation

6.1 Gary Froyland

6.1.1 Topics in integer programming and combinatorial optimisation.

Integer programming is a mathematical framework for solving large decision problems. Usually there is some underlying discrete structure for the problem such as a network or graph. You will learn new mathematical techniques in discrete mathematics, algebra, and geometry. If desired, application areas may include public or private transport modes in urban environments, medical imaging, scheduling airlines, rail, or mining processes.

6.1.2 Stochastic integer programming.

Almost all real world models have significant uncertainty in their measured data. A naive approach is to replace probability distributions of data with their mean value and create a single deterministic model. However, optimising this deterministic model typically results in decisions that are far from optimal. In order to make better decisions, the underlying probability distributions must be properly incorporated into the optimisation process. This is the aim of stochastic programming. The aim of this project is to develop rigorous optimization methods that include uncertainties in the forecast data and evaluate all possible options in light of the latest information. Familiarity with probability theory is essential. If desired, application areas may include scheduling airlines, rail, traffic, or mining processes.

6.1.3 Nonlinear and mixed integer linear optimization with application to radiotherapy.

The clinical aim of this project is to reduce imaging dose, or alternatively improve image quality, in radiotherapy treatments for lung cancer when imaging the thorax or upper abdomen using a technique known as four dimensional cone beam computed tomography. For the same image quality, we aim to reduce imaging dose by at least 50%. The mathematical component of this project involves scheduling of the 4D cone beams, taking into account a variety of geometric constraints, so as to achieve a good combination of image quality and imaging dose, and will require mathematical research in nonlinear, integer linear, and possibly integer nonlinear, optimization. (Part of a Cancer Australia Priority-driven Collaborative Cancer Research Scheme, Investigators: R. O'Brien (Medicine - Radiation Physics, USydney), G. Froyland (Mathematics, UNSW), and J.-J. Sonke (Netherlands Cancer Institute): "Reducing Thoracic Imaging Dose and Improving Image Quality in Radiotherapy Treatments"

6.1.4 Optimising fluid mixing.

Combining techniques from dynamical systems and optimisation, this project aims to develop new mathematical algorithms and practical strategies for enhancing or controlling mixing in fluids, with applications in environmental (e.g. biology or pollution) and industrial settings. The project will use mathematics from optimisation, dynamical systems, functional analysis, and probability.

6.2 Vaithilingam Jeyakumar

6.2.1 Multi-objective Optimization under Data Uncertainty.

Multi-objective optimization is the process of simultaneously optimizing two or more conflicting objectives subject to constraints. It is central to making complex management and technical decisions in industry, commerce and scientific disciplines, where optimal decisions need to be taken in the presence of trade-offs between two or more conflicting objectives. Traditional multi-objective optimization techniques assume perfect information (that is, accurate values for the input quantities or system parameters), despite the reality that such precise knowledge is rarely available in practice for real-world multi-objective optimization problems. The data of real-world multi- objective optimization problems more often than not are uncertain (that is, they are not known exactly at the time of the decision) due to estimation errors, prediction errors or lack of information.

The project will examine various mathematical principles and approaches to identifying and locating uncertainty immunized solutions of multi-objective optimization problems in the face of data uncertainty. Application areas may include multi-objective optimization problems in finance such as portfolio management problems under data uncertainty.

6.2.2 Robust optimization and data mining.

In many real world problems, the data associated with the underlying optimization problem are often uncertain due to modelling errors. Various techniques, such as stochastic programming and scenario optimization, have been developed to address these optimization problems under uncertainty. Robust optimization, which is based on a description of uncertainty by sets, instead of probability distributions, is emerging as a powerful methodology to examine uncertain optimization problems. This project will examine various robust optimization approaches to solving uncertain optimization problems. Application areas may include data mining and machine learning.

6.2.3 Semi-algebraic geometry and polynomial optimization.

What has algebraic geometry to do with optimization? The answer is: quite a lot. And all this is due to innovative ideas and links discovered in the last decade between pure and applied mathematics. A good understanding of convex sets in algebraic geometry will lead to insights into solving hard optimization problems involving polynomials. This project will examine emerging applications of algebraic geometry to optimization over polynomials.

6.2.4 Semi-Algebraic Optimization and Diffusion Tensor Imaging

The aim of this project is to examine nonlinear optimization models that arise in imaging sciences. A specific aim is to apply semi-algebraic optimization approaches to improve image quality in higher-order diffusion tensor imaging (DTI). It is a recently developed variation of magnetic resonance imaging for examining the microstructure of fibrous nerve and muscle tissue and it now allows scientists and clinicians to examine the brain in ways they hadn't been able to before. DTI is used, for example, to demonstrate subtle abnormalities in a variety of diseases including stroke, multiple sclerosis, dyslexia, and schizophrenia

In DTI, a higher-order tensor provides a mathematical tool to model and analyze the complex multi-relational clinical data. A higher-order tensor is a multi-dimensional generalization of a matrix in linear algebra to multi-way arrays. Multi-extremal semi-algebraic optimization, particularly, polynomial optimization, has proved to be a powerful methodology for higher-order DTI. The project will use mathematics from multi-linear algebra, semi-algebraic geometry, tensors and polynomial optimization.

6.3 Guoyin Li

6.3.1 Rank optimisation problem.

Notions such as order, complexity, or dimensionality can often be expressed by means of the rank of an appropriate matrix. Therefore, many practical problems can be modelled as rank optimisation problems with matrix variables. Typical examples include the matrix completion problem, minimum-order linear system realisation problem and image compression. The rank optimisation problem can be regarded as an extension of the celebrated compress sensing problem, and is often hard to analyse. One of the major difficulties in solving the rank optimisation problem is the non-smoothness and non-convexity of the rank function. In this project, we will first examine the fundamental mathematical aspects of rank functions using tools from non-smooth optimisation. We will then develop computationally efficient techniques for solving rank optimisation problems. Finally, we will apply these results to solve important problems such as the matrix completion problem and the minimum-order linear system realisation problem.

6.3.2 Optimisation approaches for tensor eigenvalue problems: modern techniques for multi-relational data analysis.

Modern data analysis is the science of correctly collecting data, assessing it for trustworthiness, extracting qualitative information from it, and presenting it in a comprehensible informative way. Traditional techniques in data analysis deal with single-relational data despite the reality that multi-relational data, whose objects have interactions among themselves based on different relations, often appear in our daily life. Examples include: researchers citing other researchers in different conferences based on different concepts and topics; papers citing other papers based on text analysis such as title, abstract, keyword and authorship; web-pages linking to other web-pages through different semantic meanings; a social network where objects are connected via multiple relations, by their organisational structure and communication protocols; biological database where laboratory experiments are carried out to understand the interactions between each individual genes and proteins in a living cell.

Mathematically, single-relational data can be modelled by a matrix. Likewise, multi-relational data can be modelled as a tensor, which is a multidimensional extension of the concept of a matrix. Solving tensor eigenvalue-eigenvector problems enable us to identify and rank the most significant factors from complex and huge-scale multi-relational data, and to express them in a way to highlight similarities and differences. In this project, we will first study the fundamental mathematical aspects of the spectral (eigenvalue) theory of tensors. We will then develop computationally efficient techniques for solving tensor eigenvalue problems using optimisation techniques. Finally, we will apply these results to problems arise in modern decision-making and image science.

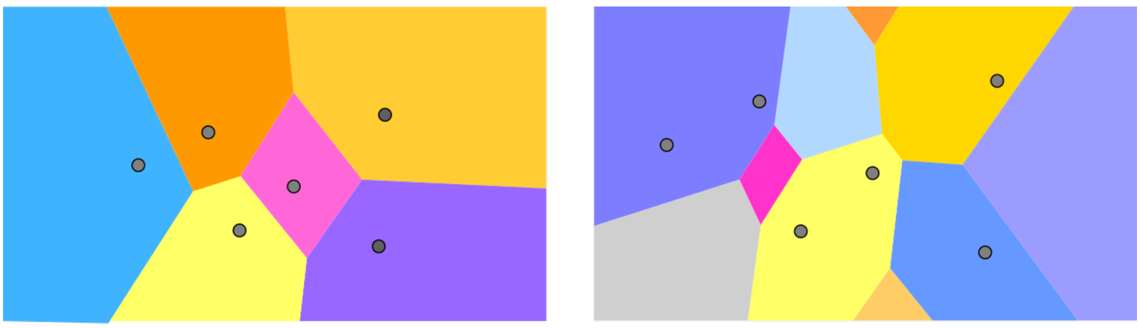
6.3.3 Nonconvex polynomial optimisation.

Many real-world problems can be modelled as non-convex polynomial optimisation problems. Due to the non-convex nature of the problem, most of the current techniques in optimisation can only find a local solution which is a relative optimiser around a given reference point. How to find a truly best solution (global optimiser) of a non-convex optimisation problem is, in general, theoretically hard and computationally challenging. In this project, we will first examine the fundamental mathematical principle for identifying a global optimiser for classes of non-convex polynomial optimisation problems. We will then develop computationally efficient techniques for locating these global optimisers. Finally, we will apply these results to solve important problems arise in engineering applications such as the sensor network localisation problem.

6.4 Vera Roshchina

6.4.1 Higher-order Voronoi cells

Classic Voronoi cells partition the space (usually the Euclidean plane) into polyhedral regions that consist of points nearest to one of the sites from a given finite set. Voronoi cells have applications in optimisation and computational geometry, such as facility location, image compression and triangulations. In this project we consider higher-order Voronoi cells that correspond to the subsets of points nearest to several sites at once. From the applied point of view, this is important for a range of problems in robust optimisation such as facility location and logistics under uncertainty. The goal is to study higher-order cells from the mathematical and computational perspectives: to determine the possible shapes of the cells and their number based on the location of sites, and to find efficient algorithms for the construction of cells from the sites and vice versa.



a) classic Voronoi cells b) two-point Voronoi cells

6.4.2 Hyperbolicity cones

A hyperbolicity cone is defined as a particular region in space bounded by the zero set (or variety) of a homogeneous polynomial with real roots along a prescribed direction. Hyperbolicity cones are convex, and have some peculiar properties. For instance, differentiating the defining polynomial produces another hyperbolicity cone. Calculating the derivative cones of hyperbolicity cones is a straightforward exercise. Here we have a tetrahedron obtained as the intersection of a four-dimensional hyperbolicity cone with a hyperplane. Differentiating its hyperbolic polynomial we obtain another hyperbolicity cone, and its three-dimensional cut corresponds to the fattened tetrahedron. In a sense it is possible to 'differentiate' a tetrahedron and obtain another convex set! Since hyperbolicity cones can be constructed by differentiating other hyperbolicity cones, a natural question is about integration. A derivative cone is obtained from differentiating the hyperbolic polynomial along a direction inside the original cone, and each direction produces a (different) hyperbolicity cone. Integration would then correspond to identifying a hyperbolic polynomial and an associated direction that produce the original hyperbolic polynomial as their derivative polynomial. This project aims at investigating the question of integration of hyperbolic polynomials and developing theoretical and numerical tools for integrating hyperbolicity cones.