

Course Outline

MATH1251 **Mathematics for Actuarial Studies and Finance**

School of Mathematics and Statistics

Faculty of Science

Semester 2, 2016

CONTENTS OF THE MATH1251 COURSE PACK 2016

Your course pack should contain the following five items:

1. *Information Booklet*
Information on administrative matters, lectures, tutorials, assessment, syllabuses, class tests, computing, special consideration and additional assessment
2. *Algebra Notes*
3. *Calculus Notes (from MATH1231/1241 as additional resource)*
4. *Calculus Problem (for MATH1251)*
Problems, answers, past class tests, table of integrals
5. *Past Exam Papers Booklet*

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GENERAL INFORMATION FOR MATH1251

MATH1251, Mathematics for Actuarial Studies and Finance 1B, is a first year 6UOC course offered by the School of Mathematics in semester 2. It is a continuation and development of the Calculus and Linear Algebra introduced in MATH1151. The prerequisite for MATH1251 is a conceded pass or better in MATH1151. The exclusions for MATH1251 are MATH1021, MATH1031, MATH1231, MATH1241, ECON1202 and ECON2291.

Contacting the Student Services Office

The School of Mathematics and Statistics web-site

<http://www.maths.unsw.edu.au>

contains many pages of useful information on mathematics courses, school policies and how to obtain help, both academic and administrative. If you cannot find the answer to your queries on the web pages you are welcome to contact the Student Services Office directly.

The first year adviser in the Student Services Office is Ms M. Lugton. All administrative enquiries concerning first year Mathematics courses should be sent to Ms Lugton, either:

- by email to fy.MathsStats@unsw.edu.au
- by phone to 9385 7011
- or in person in room RC-3072

Change of tutorials, due to timetable clashes or work commitments, permission to take class tests outside your scheduled tutorial, advice on course selection and other administrative matters are handled in the Student Services Office.

Lectures

There are two algebra lectures and two calculus lectures per week.

Lectures commence in week 1 and run until week 12 as indicated in your timetable on myUNSW. Please check your myUNSW timetable for times and locations of lectures.

The lecturers for MATH1251 are:

Algebra Dr Denis Potapov, Room 6111, Red Centre, phone 9385 7003.

Calculus Assoc. Professor Thanh Tran, Room 4061, Red Centre, phone 9385 7041.

The course authority for MATH1251 is the Director of First Year Studies, Jonathan Kress, who can be contacted via email.

The lecturer in charge of computing is Dr Jonathan Kress, Room 3073 in the Red Centre.

Important announcements and handouts may be given out in lectures, so missing lectures (or even arriving late) may cause significant difficulties for you.

Tutorials

Students in are enrolled in two tutorials, one for algebra and one for calculus. The algebra tutorial is timetabled for the second half of the week, whilst the calculus tutorial is scheduled for the first half of the week. Students are able to change their tutorials, via myUNSW, until the end of week 1, and after that time, they can only change their tutorials with the agreement of the Student Services Office, RC-3072. To change a tutorial you will need to provide proof of a timetable clash or work commitments.

Note that

- **ALL tutorials commence in week 2 and run until week 13;**
- attendance at tutorials is compulsory and the roll will be called in tutorials;

Computing and self-paced online modules

In addition to the Calculus and Algebra components, there is a Computing component in MATH1251. This is partly interwoven with the Calculus and Algebra components and partly independent of them. To assist in the self-directed learning of this component of the course, online self-paced learning modules are available in UNSW Moodle. These modules guide students through the computing component of this course and are integrated with, and enhance the lecture and tutorial content presented in Calculus and Algebra.

Students are expected to work through and complete the specified online modules.

Associated with each module is a graded quiz, done in Maple TA, and the completed quizzes contribute 4% to the final grade. Learning content will be accessible at all times for learning and revision, but the online assessments will only be available for credit until the published deadlines.

The two booklets *Computing Laboratories Information for Students 2016* and *Introduction to MATLAB 2016* are freely available from the MATH1251 module on UNSW Moodle, and also on the computers in the mathematics computing laboratories.

UNSW Moodle

The School of Mathematics and Statistics is participating in a trial of a Learning Management System called Moodle and this semester MATH1251 will use this system instead of UNSW Blackboard. To log in to Moodle use you zID and zPass at the following URL:

<http://moodle.telt.unsw.edu.au>

Once logged in, you should see a link to MATH1251 that will take you to the MATH1251 homepage in Moodle.

Assessment

The final raw mark will be made up as follows:

Algebra and Calculus class tests	20%
Online Algebra and Calculus tests	4%
Online Computing tests (MATLAB)	4%
Laboratory Computing test (MATLAB)	8%
End of semester exam	64%

Note that:

- You will **not** be allowed to take a calculator into class tests.
- Tutors are expected to enter class test marks into the School's database within a fortnight of the test being sat. These marks are then available to you through the Student Web Portal accessed via the "Maths and stats marks" link on the home page of MATH1251 on the UNSW Moodle server.

It is **your responsibility** to check that these marks are correct and you should **keep marked tests until the end of semester** in case an error has been made in recording the marks. If there is an error, either speak to your tutor or bring your test paper to the Student Services Office as soon as possible but no later than

The web page

<https://student.unsw.edu.au/exams>

has many useful links related to the running of UNSW examinations.

- As from S1, 2016 students with a final mark in the range of 45–49 will be permitted to take the Additional Assessment Exam as a Concessional Additional Assessment (AA). There will be no notification to the individual student of the right to take the Concessional AA, but the details of the courses AA exam schedule will be provided on the School's website Notice Board, after the Provisional Results are published (normally 1 week after the exam period ends). The final mark after completing the Concessional AA will not increase to a mark higher than 50. Website to School Notice Board:

<http://www.maths.unsw.edu.au/currentstudents/current-students>

- **Medical certificates will generally not be accepted for missing the deadlines for the online tests.**

Online Algebra and Calculus tests

Before each algebra and calculus tutorial class tests you must complete a simple online test that is designed to help you prepare for the tutorial tests. These tests are conducted using an online testing system called "Maple TA". Information on how to register for the correct class in Maple TA and how to use the system is provided on Moodle. To access this material and to find the link to UNSW's Maple TA server, follow the "Maple TA" link on the Moodle homepage of your first year mathematics course. Log in using your zID (z followed by your UNSW student number) as your "User login" and your zPass as the "Password". You are advised to read the instructions on Moodle before going directly to Maple TA so that you are aware of how you are expected to answer the questions. Note that you will need to self-register for the MATH1251 class in Maple TA and complete the "Declaration" and "Using Maple TA" tests before you can access tests that count for marks.

The schedule for these online tests is given below.

Test	Available	Due
TP1 - FinMath 1B Calculus online test 1	2pm Wednesday Week 3	4pm Wednesday Week 4
TP2 - FinMath 1B Algebra online test 1	2pm Monday Week 5	4pm Monday Week 6
TP3 - FinMath 1B Calculus online test 2	2pm Wednesday Week 7	4pm Wednesday Week 8
TP4 - FinMath 1B Algebra online test 2	2pm Monday Week 11	4pm Monday Week 12

To give you some familiarity with the online testing system a practice test called “Using Maple TA” will be available from week 1. You must pass this test before you will be allowed access to algebra and calculus online tests.

You will be allowed 5 attempts at each online algebra and calculus test but only your best mark for each test will count. Then, the best 3 of these 4 marks, one from each online test, will contribute up to 4% of your final grade.

Note:

- the first test becomes available at 2pm on Wednesday of week 3;
- each attempt at these tests must be your own work, but you are encouraged to discuss the methods required with other students;
- each version of a test will be slightly different, so don't just copy answers from one attempt to the next;
- only a limited numbers of users can have simultaneous access to Maple TA, so **do NOT** leave your attempts at these tests to the last day. Problems with your own (home) computer, internet service or the UNSW IT systems are not considered to be an excuse for missing tests or test deadlines.

Class tests

Details of the dates and content of tests are given on pages 16 and 19 of this booklet.

Sample copies of the tests are included in the Algebra Notes and in the Calculus problem booklet.

Sample copies of the tests are included in the Algebra and Calculus Notes.

Note that

- You **MUST** be enrolled in an Algebra tutorial and a Calculus tutorial and **YOU MUST TAKE EACH TEST IN THE TUTORIAL TO WHICH YOU HAVE BEEN OFFICIALLY ALLOCATED.**
- To each test you must bring
 - your **Student ID** card
 - some blank A4 writing paper
 - a **stapler** (so that you can staple a cover sheet to your answers).
- Normal exam conditions apply in tests.
- You will **not** be allowed to use a calculator in class tests.

- Your **best three scores** in the four tests will be counted towards your final assessment mark.
- If you are absent from a class test due to illness please **DO NOT** apply for special consideration on-line. You should take the appropriate documentation to your tutor as soon as is practicable and an M will be recorded.

Computing Assessment

There will be two different forms of computing tests. An initial set of four small online tests will be run using Maple TA, followed by a laboratory based test in week 10. The online tests may be completed on any suitable web browser in your own time, but as the MATLAB package will be needed to answer the questions, the School computing labs are probably the best place to attempt the tests. These online MATLAB computing tests are linked to the self-paced MATLAB instruction modules in UNSW Moodle. These online MATLAB computing tests will be available (almost) continuously, as they must be completed in sequence, but to gain marks for the computing component of the course the tests must be completed before the deadlines indicated below. You will have an unlimited number of attempts at these online **computing** tests, both before and after the deadlines in the following table. Note that it is only your best mark on each test that counts towards your final grade. Again, **do NOT** leave your attempts at these online tests until the last day. Inability to complete these online tests due to congestion in the school computing labs or in Maple TA on the last day will **NOT** be accepted as an excuse for missing the deadlines.

Tests	Due to be completed by
1 and 2	4pm Tuesday of week 5
3 and 4	4pm Tuesday of week 7

The online MATLAB instruction modules are numbered from 0 to 10. In Math1151 you completed the online tests for modules 1 to 6; for MATH1251 you must complete the online tests in Maple TA for modules 7 to 10. In the table above, Tests 1, 2, 3 and 4 cover the MATLAB modules 7, 8, 9 and 10 respectively. As in MATH1151, these online tests are designed to get you used to using MATLAB for simple problems and will test your knowledge of MATLAB syntax.

The second form of computing test will be run under exam conditions in the School's laboratories.

The format of the laboratory test will be similar to the one you took in MATH1151, but the content will be more challenging with more emphasis on the programming features in MATLAB. Details of the laboratory test are given on page 28 and practice problems for the test are given on page 26.

All computing tests are linked to the Algebra and Calculus material, so you should make sure you understand the course work before trying them.

Finally, the end of semester exam may contain one or two sub-questions requiring a knowledge of MATLAB.

SCHEDULE OF ALL CLASS ASSESSMENTS

The table below gives the schedule of online tests, class tests and computing assessments.

Week	Algebra	Calculus	MATLAB Computing
1			
2			
3			
4		TP1	
5		Test 1	Online tests 1 and 2 due
6	TP2, Test 1		
7			Online tests 3 and 4 due
8		TP3	
9		Test 2	
Mid-semester break			
10			Test in Laboratory
11			
12	TP4, Test 2		
13			
End of semester examination — check UNSW exam timetables for details			

Course Materials

The course materials for MATH1251 are:

MATH1251 *Course Pack 2016*.

Computing Laboratories Information 2016 and *Introduction to MATLAB 2016*.

S.L. Salas, E. Hille and G.J. Etgen, *Calculus - One and Several Variables*, any recent edition, Wiley.

The latest edition of the textbook, Salas, Hille and Etgen *Calculus - One and Several Variables*, 10th Edition comes packaged with access to the electronic resources known as WileyPlus. This electronic version provides internet access to the textbook, problems, worked solutions, tests (for self-assessment) and other electronic resources related to the text material. The purchase of the text from the UNSW Bookshop gives web access to the WileyPlus server for one year; it is possible to renew the web access on a yearly basis at a fee determined by the publisher. It is also possible to purchase just the web access to the electronic version of the textbook for one year. This can also be done at the UNSW Bookshop. Note that these WileyPlus electronic resources are provided by the publisher John Wiley, and **not** by the School of Mathematics and Statistics. Any difficulties that you might have with access to WileyPlus must be resolved directly with the publisher.

Salas & Hille is sold at the UNSW Bookshop.

Course Packs are also sold through the UNSW Bookshop, while the computing laboratory notes and introduction to MATLAB are free to download.

The Course Pack contains the following items:

- *Information Booklet that you are now reading;*
- *Algebra Notes (for MATH1251);*
- *Calculus Notes (from MATH1231/1241 as an additional resource);*
- *Calculus Problems Booklet;*
- *Past Exam Papers Booklet.*

For the computing component of the course you will need to use Matlab. This is available for free to UNSW students. You can either download and installed on your own computer or use the virtual Matlab application via the myAccess service. Details will be provided on Moodle.

The book

R. Pratap, *Getting Started with MATLAB Version 6*, Oxford University Press, 2002

is available on short-term loan from RC-3072.

Booklets contained in the Course Pack will **not** be available separately from the School of Mathematics and Statistics. However the information in this booklet and the algebra and calculus problems can be accessed through the web from the MATH1251 module on the UNSW Moodle server.

Getting help outside tutorials

There will be a roster which shows for each hour of the week a list of names of members of staff who are available at that time to help students in first year mathematics courses. This roster is displayed on the noticeboard near the School Office (Room 3070, Red Centre) and outside the Student Services Office (Room 3072, Red Centre). It is also available from the web page

<http://www.maths.unsw.edu.au/currentstudents/consultation-mathematics-staff>

Graduate Attributes

MATH1251 will provide you with an in-depth knowledge of topics in Calculus and Linear Algebra and show applications in interdisciplinary contexts through lectures and exercises. It will enhance your skills in analytical critical thinking and problem solving through illustrative examples in lectures and problem based tutorials. The course will also engage you in independent and reflective learning through your independent mastery of tutorial problems and MATLAB. The mathematical problem solving skills that you will develop are generic problem solving skills, base on logical arguments, that can be applied in multidisciplinary work. You will develop your communication skills through active participation in tutorials, and by writing clear, logical arguments when solving problems.

Academic misconduct

It is very important that you understand the University's Rules for the conduct of Examinations and the penalties for Academic Misconduct Guide. This information can be accessed through myUNSW at:

<https://student.unsw.edu.au/exams>.

Illness and other problems

If your performance in this course is affected by illness or other serious difficulties which are beyond your control, you can apply for Special Consideration and you may be offered the opportunity for Additional Assessment. You should also seek advice from the Director of First Year, Dr Jonathan Kress.

In order to be offered Additional Assessment it is essential that you **follow exactly the procedures set out in the document entitled “Application for Special Consideration in MATH1231, MATH1241 and MATH1251 2016”**.

A copy of this document is included in this booklet on page 10.

Take particular note that

- The School will **NOT** contact you to tell you that you have been granted Additional Assessment. It is **YOUR RESPONSIBILITY** to find this out by following the instructions in the document mentioned above.
- **If you have a poor record of attendance or performance during the semester you may be failed regardless of illness or compassionate grounds affecting the final exam.**

Note also that

- If illness affects your attendance at or performance in a **class test**, do **not** make an application for Special Consideration. Simply show the original medical certificate to your tutor and also give a copy of the medical certificate to your tutor. This information will be taken into account when calculating your final assessment mark.
- Transport delays and oversleeping will **not** be accepted as reasons for missing class tests. (But note that only your best three test results are counted for assessment.)
- Because it is possible to sit the laboratory computing test on many days, **except in unusual circumstances, medicals will not be accepted as excuses for not sitting the computing test.** Therefore, it is recommended that you book to sit a laboratory test at an early time.
- If you arrive too late to be admitted to the end of semester exam, go **immediately** to the Mathematics and Statistics Student Services Office, Room 3088, Red Centre.

School of Mathematics and Statistics Policies

Students in courses run by the School of Mathematics and Statistics should be aware of the School and Course policies by reading the appropriate pages on the MathsStats web site starting at:

<http://www.maths.unsw.edu.au/currentstudents/assessment-policies>

**APPLICATIONS FOR SPECIAL CONSIDERATION IN
MATH1231, MATH1241 AND MATH1251 SEMESTER 2 2015**

If you feel that your performance in, or attendance at, a final examination has been affected by illness or circumstances beyond your control, or if you missed the examination because of illness or other compelling reasons, you may apply for special consideration. Such an application **may** lead to the granting of additional assessment.

It is essential that you take note of the rules 1, 2, 5 and 6, which apply to applications for special consideration in all first year Mathematics courses. Rules 3 and 4 apply to the above courses only.

1. **Within 3 days** of the affected examination, or at least as soon as possible, you must **submit a request for special consideration to UNSW Student Central ON-LINE**.

Please refer to link below for How to Apply for Special Consideration,

<https://student.unsw.edu.au/special-consideration>

2. **Please do not expect an immediate response from the School.** All applications will be considered together. See the information below.
3. If you miss a **class test** due to illness or other problems, then you should provide the appropriate documentation to your **tutor** who will record an M. **DO NOT** apply on-line for special consideration for class tests or for on-line or computing tests.
4. If your course involves a MAPLE/MATLAB lab test which you miss, you should contact the lecturer in charge of computing as soon as possible. A resit will be organised for later in the session.
5. **You will NOT be granted additional assessment in a course if your performance in the course** (judged by attendance, class tests, assignments and examinations) **does not meet a minimal standard**. A total mark of greater than 40% on all assessment not affected by a request for special consideration will normally be regarded as the minimal standard for award of additional assessment.
6. It is **YOUR RESPONSIBILITY** to find out **FROM THE SCHOOL OF MATHEMATICS AND STATISTICS** whether you have been granted additional assessment and when and where the additional assessment examinations will be held. **Do NOT wait to receive official results from the university**, as these results are not normally available until after the Mathematics additional assessment exams have started.

Information about award of Additional Assessment and a provisional list of results will be made available on the Maths & Stats Marks page late on Friday 25th of November. A link to the Maths & Stats Marks page is provided on Moodle.

7. The Additional Assessment exams for MATH1131, MATH1231, MATH1241 and MATH1251 will be on Tuesday 29th November. A link to the Additional Assessment timetable, including locations, will be placed on the Current Students Notice Board under heading "Special Consideration and Additional Assessment" information which can be found at:

<http://www.maths.unsw.edu.au/currentstudents/current-students>

8. If you have two additional assessment examinations scheduled for the same time, please consult the School of Mathematics and Statistics Office as soon as possible so that special arrangements can be made.

9. You will need to produce your UNSW Student Card to gain entry to additional assessment examinations.

IMPORTANT NOTES

- The additional assessment examination may be of a different form from the original examination and must be expected to be at least as difficult.
- If you believe that your application for special consideration has not been processed, you should immediately consult the Director of First Year Studies of the School of Mathematics and Statistics (Room 3073 Red Centre).
- If you believe that the above arrangements put you at a substantial disadvantage, you should, at the earliest possible time, send full documentation of the circumstances to the Director of First Year Studies, School of Mathematics and Statistics, University of New South Wales, Sydney, 2052.
- If you suffer from a chronic or ongoing illness that has, or is likely to, put you at a serious disadvantage, then you should contact the Disability Support Services who provide confidential support and advice. Their web site is:

<https://student.unsw.edu.au/disability>

Disability Support Services (DSS) may determine that your condition requires special arrangements for assessment tasks. Once the School has been notified of these we will make every effort to meet the arrangements specified by DSS.

- Additionally, if you have suffered misadventure during semester then you should provide full documentation to the Director of First Year Mathematics as soon as possible. In these circumstances it may be possible to arrange discontinuation without failure or to make special examination arrangements.

Professor B. Henry
Head, School of Mathematics and Statistics

UNIVERSITY STATEMENT ON PLAGIARISM

Plagiarism is the presentation of the thoughts or work of another as one's own.¹ Examples include:

- direct duplication of the thoughts or work of another, including by copying work, or knowingly permitting it to be copied. This includes copying material, ideas or concepts from a book, article, report or other written document (whether published or unpublished), composition, artwork, design, drawing, circuitry, computer program or software, web site, Internet, other electronic resource, or another person's assignment without appropriate acknowledgement
- paraphrasing another person's work with very minor changes keeping the meaning, form and/or progression of ideas of the original;
- piecing together sections of the work of others into a new whole;
- presenting an assessment item as independent work when it has been produced in whole or part in collusion with other people, for example, another student or a tutor; and,
- claiming credit for a proportion a work contributed to a group assessment item that is greater than that actually contributed².

Submitting an assessment item that has already been submitted for academic credit elsewhere may also be considered plagiarism.

The inclusion of the thoughts or work of another with attribution appropriate to the academic discipline does not amount to plagiarism.

Students are reminded of their Rights and Responsibilities in respect of plagiarism, as set out in the University Undergraduate and Postgraduate Handbooks, and are encouraged to seek advice from academic staff whenever necessary to ensure they avoid plagiarism in all its forms.

The Learning Centre website is the central University online resource for staff and student information on plagiarism and academic honesty. It can be located at:

www.lc.unsw.edu.au/plagiarism

The Learning Centre also provides substantial educational written materials, workshops, and tutorials to aid students, for example, in:

- correct referencing practices;
- paraphrasing, summarising, essay writing, and time management;
- appropriate use of, and attribution for, a range of materials including text, images, formulae and concepts.

Individual assistance is available on request from The Learning Centre.

Students are also reminded that careful time management is an important part of study and one of the identified causes of plagiarism is poor time management. Students should allow sufficient time for research, drafting, and the proper referencing of sources in preparing all assessment items.

¹Based on that proposed to the University of Newcastle by the St James Ethics Centre. Used with kind permission from the University of Newcastle.

²Adapted with kind permission from the University of Melbourne

ALGEBRA SYLLABUS AND LECTURE TIMETABLE

The algebra course for MATH1251 is based on chapters 6 to 9 of the MATH1251 Algebra Notes, which are essential reading and must be brought to all algebra tutorials. The lecturer will not cover all the material in these notes in their lectures as some sections of the notes are intended for reference and for background reading. An **approximate** lecture timetable is given below. The lecturer will try to keep to this timetable, but variations might be unavoidable. As in MATH1151, the computer package MATLAB will be used in the MATH1251 algebra course.

Chapter 6. Complex Numbers

Lectures 1–6.

This section covers complex numbers in Cartesian and polar forms, complex arithmetic and geometry, factorization of polynomials and stability of dynamical systems.

Development of number systems and closure. Definition of complex numbers and of complex number addition, subtraction, multiplication and division.

Equality, real and imaginary parts, complex conjugates.

Argand diagram, polar form, modulus, argument.

De Moivre's Theorem and Euler's Formula. Arithmetic of polar forms.

Powers and roots of complex numbers. Binomial theorem and Pascal's triangle.

Trigonometry and geometry.

Complex polynomials. Fundamental theorem of algebra, factorization theorem, factorization of complex polynomials of form $z^n - z_0$, real linear and quadratic factors of real polynomials.

Stability of discrete and continuous time systems.

Chapter 7. Vector Spaces

Lectures 7–14

The aim of this section of the course is to introduce the general theory of vector spaces and to give some basic examples. The majority of examples will be for the real vector space \mathbb{R}^n , but some examples will be given for the complex vector space \mathbb{C}^n , the vector space M_{mn} of $m \times n$ matrices, the vector space of polynomials and the vector space of real-valued functions.

Introduction to vector spaces. Examples of vector spaces. Properties of vector arithmetic.

Subspaces.

Linear combinations and spans.

Linear independence.

Basis and dimension, coordinate vectors.

Polynomials and real-valued functions as vector spaces.

Data fitting and polynomial (Lagrange) interpolation.

Chapter 8. Linear Transformations

Lectures 15–18

The basic aims of this section are to introduce the general theory of linear transformations, to give some geometric applications of linear transformations and to establish the close relationship between linear functions and matrices.

Introduction to linear maps. Linear maps and the matrix equation.

Geometrical examples.

Subspaces associated with linear maps.

Rank, nullity and solutions of $A\mathbf{x} = \mathbf{b}$. Further applications.

Linear maps between polynomial and real-valued function vector spaces.
Matrix representations for non-standard bases in domain and codomain.
Matrix arithmetic and linear maps.
Injective, surjective and bijective linear maps.

Chapter 9. Eigenvalues and Eigenvectors

Lectures 19–24

The aims of this section are to introduce the ideas of eigenvalue and eigenvector and to show some applications of these ideas to diagonalization of matrices, evaluation of powers of matrices and solution of simple systems of linear differential equations. Examples for hand calculation will be restricted to 2×2 matrices and very simple 3×3 matrices, with larger problems done using MATLAB.

Definition, examples and geometric interpretation of eigenvalues and eigenvectors.

Eigenvectors, bases and diagonalization of matrices.

Applications to powers of matrices and solution of systems of linear differential equations.

Markov Chain Processes.

PROBLEM SETS

At the end of each chapter there is a set of problems. Some of the problems are very easy, some are less easy but still routine and some are quite hard. To help you decide which problems to try first, each problem is marked with an **[R]**, an **[H]** or an **[X]**. The problems marked **[R]** form a basic set of problems which you should try first. Problems marked **[H]** are harder and can be left until you have done the problems marked **[R]**. You *do* need to make an attempt at the **[H]** problems because problems of this type will occur on tests and in the exam. If you have difficulty with the **[H]** problems, ask for help in your tutorial.

The problems marked **[X]** are extension material and are intended for students aiming for the grade of HD in MATH1251 – they require more mathematical maturity and ingenuity than other suggested problems. Small parts of final exam questions may require similar mathematical understanding.

There are a number of questions marked **[M]**, indicating that MATLAB is required in the solution of the problem.

PROBLEM SCHEDULE

The main purpose of tutorials is to give you an opportunity to get help with problems which you have found difficult and with parts of the lectures or the Algebra Notes which you don't understand. In order to get real benefit from tutorials, it is essential that you try to do relevant problems *before* the tutorial, so that you can find out the areas where you need help. The following table shows the calculus problems which are relevant to each week's calculus tutorial. You should work on them at home or in the library between classes. Some of them will be worked through and discussed in the tutorials. Tutors may need to vary a little from this suggested problem schedule.

Week	Algebra problems
1	No tutorial, but try the revision questions
2	Chapter 6, 1–28
3	Chapter 6, 29–59
4	Chapter 6, 60–84
5	Chapter 7, 1–33
6	Chapter 7, 34–52 (Test 1)
7	Chapter 7, 53–82
8	Chapter 7, 83–104
9	Chapter 8, 1–23
10	Chapter 8, 24–43
11	Chapter 8, 44–59
12	Chapter 9, 60–72
	Chapter 9, 1–10 (Test 2)
13	Chapter 9, 11–34

CLASS TESTS AND EXAMS

Questions for the class tests in MATH1251 will be similar to the questions marked **[R]** and **[H]** in the problem sets. Since each class test is only twenty minutes in length only shorter straight forward tests of theory and practice will be set. As a guide, see the recent past class test papers (at the end of the Algebra notes).

Examination questions are, by their nature, different from short test questions. They may test a greater depth of understanding. The questions will be longer, and sections of the course not covered in the class tests will be examined. As a guide, see the recent past exam papers in the separate past exam papers booklet.

Algebra class test 1 will be given in week 6 and will be based on the suggested problems from for weeks 2 to 5.

Algebra class test 2 will be given in week 12 and will be based on the suggested problems for weeks 6 to 11.

THEORY IN THE ALGEBRA COURSE

The theory is regarded as an essential part of this course and it will be examined both in class tests and in the end of year examination.

You should make sure that you can give DEFINITIONS of the following ideas:

Chapter 7. Subspace of a vector space, linear combination of a set of vectors, span of a set of vectors, linear independence of a set of vectors, spanning set for a vector space, basis for a vector space, dimension of a vector space, coordinate vector of a vector with respect to an ordered basis.

Chapter 8. Linear function, kernel and nullity of a linear function, image and rank of a linear function.

Chapter 9. Eigenvalue and eigenvector, diagonalizable matrix.

You should be able to give STATEMENTS of the following theorems and propositions.

Chapter 7. Theorem 1 of §7.3, Propositions 1 and 3 and Theorem 2 of §7.4, Proposition 1 and Theorems 2, 3, 4, 5 and 6 of §7.5, Theorems 1, 2, 3, 4, 5, 6 and 7 of §7.6.

Chapter 8. Theorems 2, 3 and 4 of §8.1, Theorems 1 and 2 of §8.2, Theorems 1 and 5, Proposition 7 and Theorems 8, 9 and 10 of §8.4.

Chapter 9. Theorems 1, 2 and 3 of §9.1, Theorem 1 and 2 of §9.2.

You should be able to give PROOFS of the following theorems and propositions.

Chapter 7. Theorem 2 of §7.4, Theorems 2, 3 and 4 of §7.5, Theorem 2 of §7.6.

Chapter 8. Theorem 2 of §8.1, Theorem 1 of §8.2, Theorems 1, 5 and 8 of §8.4.

Chapter 9. Theorem 1 of §9.1.

CALCULUS SYLLABUS FOR MATH1251

The calculus syllabus below contains a number of topics which are part of the MATH1241 syllabus and in addition some topics that are generally taught to second year students – such as Lagrange multipliers and double integrals. The time given for each topic is approximate.

1. Integration techniques (2 hours)

Trigonometric integrals and reduction formulae, trigonometric and hyperbolic substitutions, rational functions and partial fractions, standard substitutions. (Salas and Hille, edition 10, Chapter 8, 8.1–8.6.)

2. Ordinary differential equations (7 hours)

Terminology including particular, general, implicit and explicit solutions. First order equations including separable, linear and exact equations. Slope fields, integral curves, existence and uniqueness. Modelling. Linear differential operators as linear transformations, the solution space for homogeneous differential equations and the solution set for inhomogeneous differential equations. Constant coefficient differential equations including the method of undetermined coefficients. Applications. Dimension of the solution space. Euler's method for the numerical solutions of differential equations. (Salas and Hille, edition 10, Chapter 9, 9.1–9.3, Chapter 19, 19.1–19.4.)

3. Taylor series (7 hours)

Approximation of functions by Taylor polynomials and Taylor's Theorem with remainder. Applications to stationary points.

Sequences: Convergence and divergence, sums, products, quotients and composites of series. Upper and lower bounds, sup and inf, bounded monotonic sequences and the completeness of the real numbers. Recursively defined sequences. (Salas and Hille, edition 10, Chapter 11, 11.1–11.3.)

Series: Partial sums, convergence and divergence, the k th term test for divergence. Comparison, integral, ratio and root tests for convergence of series with positive terms. Absolute and conditional convergence, including the alternating series (Leibniz') test and rearrangement considerations. (Salas and Hille, edition 10, Chapter 12, 12.1–12.5.)

Power series: Taylor and Maclaurin series. The radius and interval of convergence. Manipulation of power series by addition, multiplication, differentiation, integration and simple substitutions. Commonly occurring power series. (Salas and Hille, edition 10, Chapter 12, 11.6–12.9.)

4. Further functions of several variables (3 hours)

Tangent planes and Taylor series for functions of two variables. Classification of critical points of functions of two variable. The method of Lagrange multipliers for constrained extrema. (Salas and Hille, Edition 10, Chapter 16, 16.4–16.7.)

5. Double integrals (4 hours)

The double integral as a repeated integral, interchange of order of integration, change of variable. (Salas and Hille, Edition 10, Chapter 17, 17.1–17.4.)

PROBLEM SETS

The Calculus problems are provided in a separate booklet in the course pack. Additional problems are located at the end of each chapter of the MATH1231 and MATH1241 Calculus Notes booklet. They are also available from the course module on the UNSW Moodle 2 server. Some of the problems are very easy, some are less easy but still routine and some are quite hard. All students should make sure that they attempt and can do the unstarred questions. The starred problems are slightly harder than the unstarred problems and the double starred problems are just plain difficult!

Remember that working through a wide range of problems is the key to success in mathematics.

PROBLEM SCHEDULE

The main reason for having tutorials is to give you a chance to get help with problems which you find difficult and with parts of the lectures or textbook which you don't understand. To get real benefit from tutorials, you need to try the relevant problems *before* the tutorial so that you can find out the areas in which you need help. The following table shows the calculus problems which are relevant to each week's calculus tutorial. You should work on them at home or in the library between classes. Some of them will be worked through and discussed in the tutorials. Tutors may need to vary a little from this suggested problem schedule.

Week	Calculus problems
1	No tutorial.
2	1–9
3	10–20
4	21–35
5	36–45 (Test 1)
6	46–59
7	60–69;
8	70–83
9	84–85, (Test 2)
10	86–95
11	96–104
12	105–116
13	117–124

Note there is some overlap between the MATH1231 questions and those in the MATH1251 calculus problems.

CLASS TESTS AND EXAMS

Questions for the class tests in MATH1251 will be similar to the unstarred and single starred questions in the MATH1251 calculus problem book, or the questions marked [R] and [H] in the MATH1231 problems. Since each class test is only twenty minutes in length only shorter straight forward tests of theory and practice will be set. As a guide, see the recent past class test papers (at the end of the Calculus problem booklet). The Calculus class tests will take place in tutorials in the following weeks:

Test 1 Week 5

Test 2 Week 9

The tests will cover sections of the syllabus as shown in the table below. The table also shows which problems are relevant to each test.

Test	Syllabus sections	Unstarred and starred problems in ranges
1	1. Integration techniques 2. Ordinary differential equations	1–13 14–35
2	2. Ordinary differential equations 3. Taylor series	36–62 MATH1231 Ch 4; 1–24

It is important to note that the class tests do not cover the whole syllabus.

Examination questions are, by their nature, different from short test questions. They may test a greater depth of understanding. The questions will be longer, and sections of the course not covered in the class tests will be examined. As a guide, see the recent past exam papers in the separate past exam papers booklet.

Aims

The aim of MATH1251 is that by the time you finish the course you should understand the concepts and techniques covered by the syllabus and have developed skills in applying those concepts and techniques to the solution of appropriate problems. Students who achieve good competence in this course should be well equipped both technically and psychologically to cope with the mathematics that they will meet later in their program.

The algebra component contains a topic on complex numbers but the main emphasis of the course is on linear algebra – vector spaces, linear transformations and eigenvectors of matrices. There are a variety of topics to be covered in the calculus section – techniques of integration, differential equations, sequences and series and the calculus of functions of two real variables. (A detailed syllabus can be found later in this booklet.) The syllabus includes a computing component, based on the software package MATLAB. The computer-based tutorial problems and assignments define the level of proficiency you are expected to achieve in using MATLAB.

Learning Outcomes

A student should be able to:

- state definitions as specified in the syllabus,
- state and prove appropriate theorems,
- explain how a theorem relates to specific examples,
- apply the concepts and techniques of the syllabus to solve appropriate problems,
- prove specific and general results given specified assumptions,
- use mathematical and other terminology appropriately to communicate information and understanding,
- use the computing package MATLAB as an aid to solve appropriate problems.

Remote Access

All of the software that you need for this course is installed on the computers in the Red-Centre labs. This software can also be accessed from your own computer. For information on accessing Mathematical and Statistical software from outside the Red-Centre labs, please see the information provided on this course's page in UNSW Moodle.

Accounts and passwords

If you had an account for computers in the Mathematics Labs in you will continue to use the same account with the same password in semester 2.

Remember that for the computers in the school laboratories, your login ID is “z” followed immediately by your seven digit student number and your password is your zPass. If you have difficulties logging in, the computers will allow a five minute login with ID “newuser” and password “newuser” where you can access <https://idm.unsw.edu.au> and reset or unlock your zPass. Be aware that two consecutive failed login attempts will lock you out of the computing system for 30 minutes, or until you reset or unlock your zPass. If you have forgotten how to log in and use the lab computers help can be found in chapters 1–4 of the Computing Laboratories Information 2016 booklet and the School web site.

If you have problems with your account, you should go to Room M022 on the Mezzanine Level of the Red Centre between 1pm and 2pm on any weekday from Thursday of Week 1. You will need to show your student card.

Learning Matlab

As a rough guide, you should spend around one hour per week on computing in MATH1251. This is an average figure, and we recommend that you make a special effort in the first few weeks to master the basics. In lectures, you will see numerous examples of how MATLAB is used to solve a variety of mathematical problems, but there is not sufficient class time for a systematic treatment of MATLAB.

When you come to write M-files (scripts or functions) you will need to use an editor. We recommend the built-in MATLAB editor (type `help edit`) because it has several features specifically tailored to writing MATLAB programs. Nevertheless, you can use any of the other available editors, such as `kwrite`, `xedit`, `axe`, `emacs` or `nedit`.

Limited help will be available from the consultants who will be available in Room G012 from 12noon to 4pm each day. These consultants may have little or no knowledge of MATLAB, so you should only expect them to assist with UNIX/X-windows difficulties.

If you have any constructive criticism or comment about the Computing component then please let us know.

Maple

The other first-year mathematics courses use a different software package called Maple. However, the Actuarial Studies Unit advised us that MATLAB was more suitable for their purposes and would be introduced into their second- and third-year courses. Many later-year applied mathematics courses — including those taken by students in Finance/Mathematics programs — already use MATLAB. Some later-year pure mathematics courses use Maple.

The main distinction between the two software packages is that whereas MATLAB works primarily with arrays of numeric data, Maple works primarily with symbolic expressions. We do

not expect you to learn Maple in MATH1251, but it is available on the PCs in the Mathematics computing labs and you are free to use it.

Matlab Toolboxes

As well as its kernel routines, MATLAB has a collection of specialised software libraries called *toolboxes*. We will not use any of them in MATH1151 or MATH1251, but in later-year courses many of you will see the financial, statistics and the optimization toolboxes. Use the MATLAB `help` command to see a complete list of the toolboxes available on the computing laboratory PCs.

One toolbox not available is the Symbolic Math Toolbox, which essentially allows you to use certain Maple commands within MATLAB.

Assessment

There will be two different forms of computing tests. The details of the online MATLAB tests have been described previously in the section on Computing tests on page 6.

The second form of computing test will be run under exam conditions in the School's computing laboratories during week 10.

You must book for the test using the lab test booking link the MATH1251 page on UNSW Moodle, and bring your UNSW student ID card to the laboratory test.

All tests are linked to the Algebra and Calculus material, so you should make sure you understand the course work before trying them.

Finally, the end of semester exam may contain one or two sub-questions requiring a knowledge of MATLAB.

Special consideration for the laboratory test

Because the computing tests can be sat at many different times, medical, or other, reasons for missing the test will generally not be accepted. For this reason you are advised to choose an early time to sit the test. If you consider that you have an exceptional reason for missing the test then you must speak to Dr Q. Le Gia, Lecturer in Charge MATH1251 Computing as soon as possible after the tests have been completed.

If you are unable to attend your Maple laboratory test session because of illness or other circumstances outside of your control you should contact the Student Services Office as soon as possible and on submission of a suitable medical certificate or other appropriate documentation, another test time will be arranged.

Tutors do not have permission to accept medical certificates for the computing test.

If possible, special arrangements for the computing laboratory test will be made for students with supporting documentation from SEADU. If you wish to exercise this option, you must contact Dr Le Gia before the laboratory tests have commenced so that any needed special facilities can be implemented.

Dr Q Le Gia (Room: Red Centre 2084)
Lecturer in Charge
MATH1251 Computing

Details of the computer laboratory Matlab test follow in the next pages.

The test will be on the features of Maple which are covered in Chapter 1 and Chapter 2 of the First Year Maple Notes 2016.

You will NOT need to remember the exact syntax of each command because you will be provided with a hard copy of the First Year Maple Notes in the test and you will also have access to an on-line copy of the Notes. However, you WILL need to practise for the test by working through the problems on the attached problem sheet. Don't just sit at home and work out commands which you think will work. It is essential that you try out your answers on the computer to check that they do work and to get practice at recognising and recovering from common mistakes such as omitting the colon in `:=` or forgetting to unassign a variable.

For each problem on the problem sheet, we have provided an answer which shows you what the final Maple output should be, but not the commands which you might use to get that answer. If you have difficulty doing one of these problems, ask for help from one of the computing consultants in the labs. **If your Maple worksheet crashes while you are working on the practice problems, please make a note of what you were doing at the time and inform one of the computing consultants.**

As the laboratory test will be conducted using Maple TA, the format of this test will be different to the previous Maple laboratory tests you may have taken. The style of question asked within the Maple TA based test will be similar to those posed in the online Maple tests associated with the Maple self-paced learning modules. You can expect the questions to be more difficult than those in the online Maple tests and closer to those in the practice problems and the sample test.

A sample laboratory test will be made available in Maple TA by the end of week 4. Try to do it in 40 minutes AFTER you have worked through all the practice problems. More details on this new form of laboratory test will be made available during the session and posted on Blackboard.

You will NOT be allowed to take any calculators or writing materials (pens, pencils, paper) into the test.

Notes and Practice Problems for the MATH1251 Matlab Computing Test

The MATH1251 Matlab Computing Test will be held in the School of Mathematics Computing Laboratory, Red Centre G012, in Week 10.

- You must book **one** time slot through the Mathematics student web portal. Go to the MATH1251 Moodle 2 page and click on the “Maths Info (Maths marks)” link. Login using your student number and zPass. If a particular time slot is full then bookings for that time will be disabled.
- Make sure you can login to the Linux computers in RC-G012 **before** the test!
- You can download any data files mentioned in the sample questions from the MATH1251 module in Moodle 2. During Week 9 you will be able to download model solutions.
- **You need to prepare for the test well in advance, because the labs will be heavily used in week 9, and the downstairs labs will be unavailable for all of week 10.**

You are expected to be familiar with the following MATLAB commands:

<code>complex</code>	defining a complex number	<code>plot</code>	2-D plot
<code>real</code>	real part	<code>title</code>	put a title on a plot
<code>imag</code>	imaginary part	<code>xlabel</code>	label the x-axis of a plot
<code>conj</code>	complex conjugate	<code>ylabel</code>	label the y-axis of a plot
<code>abs</code>	modulus	<code>grid</code>	add grid lines to a plot
<code>angle</code>	argument	<code>disp</code>	display simple text messages
<code>[]</code>	creating vectors and matrices	<code>if</code>	conditional statement
<code>:</code>	colon operator	<code>num2str</code>	convert value to a string
<code>rref</code>	reduced row-echelon form	<code>print -dps</code>	save a plot as a postscript file
<code>eye</code>	identity matrix	<code>function</code>	first line of a function
<code>\</code>	matrix left-division operator	<code>%</code>	comments
<code>A'</code>	transpose A^T of A	<code>clear</code>	clear variables from workspace
<code>linspace</code>	equally spaced points	<code>what</code>	list Matlab files in current directory
<code>roots</code>	zeros of a polynomial	<code>who</code>	list variables in workspace
<code>polyval</code>	evaluate a polynomial	<code>polyfit</code>	polynomial fit to data
<code>ode23</code>	solve a 1st-order ODE	<code>load</code>	load data from a file
<code>@(x)</code>	anonymous function	<code>meshgrid</code>	xy -grids
<code>surf</code>	surface plot	<code>contour</code>	contour plot
<code>colorbar</code>	colour key		

As for MATH1151, you need to understand how to create and modify m-files using the MATLAB editor. You should also know how to obtain help using the electronic documentation provided by MATLAB.

The harder questions are marked with an asterisk *. Each test will contain four routine questions and one harder one. The test will be 45 minutes long.

1. Define the complex number $z = -1 + 3i$ and calculate

$$v = \frac{(3 + 2i)z^{12}}{(\bar{z} - 4i)^4}.$$

Find the modulus and argument of v , and store the modulus in the variable `vmod` and the argument in the variable `varg`.

2. Calculate a complex vector \mathbf{w} containing all the 12th roots of $5 - 8i$ and plot them on an Argand diagram. Label the axes, add a grid and a title and then print the plot to a postscript file called `myplot.ps`.
3. Calculate the complex vector \mathbf{w} with $w_j = z^j$ for $j = 1, 2, \dots, 20$ and $z = \frac{1}{2} - \frac{2}{3}i$. Plot the components of \mathbf{w} on an Argand diagram as stars `*` joined by straight lines, label the real and imaginary axes and add a grid. Print the figure to a postscript file called `myplot.ps`.
4. Calculate the complex vector \mathbf{w} with $w_j = e^{zt_j}$ for $t_j = 0, 0.1, 0.2, \dots, 9.9, 10$ and $z = -\frac{2}{3} + \frac{3}{4}i$. Plot the components of \mathbf{w} , joined by lines, on an Argand diagram, label the real and imaginary axes and add a grid. Print the figure to a postscript file called `myplot.ps`.
5. Let $S = \{(-2, 4, -6, 8), (1, -6, 15, -16), (5, -8, 9, -14), (-4, -2, 18, -14)\}$. Determine if the following vectors are in $\text{span}(S)$.

(a) $\mathbf{b}_1 = (0, -2, 6, -6)$.

(b) $\mathbf{b}_2 = (1, 1, 1, 1)$.

Display one of the text messages `b1 is in the span of S` or `b1 is not in the span of S`. This can be done just by inspecting the appropriate row echelon form.

Note that the set S may be linearly independent or linearly dependent.

6. Determine if the following sets of vector are linearly independent or linearly dependent. Store the vectors as `u1, u2, ...`, and display a message `Vectors are linearly dependent` or `Vectors are linearly independent`.
 - (a) $\{(1, -3, 4, 2), (4, 3, 9, -4), (4, 0, -3, 2)\}$;
 - (b) $\{(-2, -9, -1, 8), (5, 0, 13, -2), (7, -9, 9, 8), (12, 0, -9, 6)\}$;

7. Find a basis for \mathbb{R}^5 containing as many of the vectors

$$\begin{aligned} \mathbf{v}_1 &= (7, -2, 3, 4, 6), \\ \mathbf{v}_2 &= (3, 0, -1, 11, 16), \\ \mathbf{v}_3 &= (51, -24, 49, 95, 9), \\ \mathbf{v}_4 &= (5, -4, 8, -14, -20), \\ \mathbf{v}_5 &= (0, 0, -1, 11, 16), \\ \mathbf{v}_6 &= (5, 11, 21, -19, 31), \end{aligned}$$

as possible. Store these vectors as **column** vectors `v1, ... , v6` and the basis as the columns of the matrix `B`.

8. Show that the vectors

$$\begin{aligned}\mathbf{u}_1 &= (1, 2, -2, 1)/\sqrt{10}, \\ \mathbf{u}_2 &= (3, 0, 0, -3)/\sqrt{18}, \\ \mathbf{u}_3 &= (0, -2, -2, 0)/\sqrt{8}, \\ \mathbf{u}_4 &= (2, -1, 1, 2)/\sqrt{10},\end{aligned}$$

are **orthonormal**. Hence solve the linear system $U\mathbf{x} = \mathbf{b}$, where $U = [\mathbf{u}_1 \ \mathbf{u}_2 \ \mathbf{u}_3 \ \mathbf{u}_4]$ and $\mathbf{b} = (1, 1, 1, 1)$, **in the most efficient way**.

9.* Write a function `w = allroots(z, n)` which calculates an n -dimensional complex vector \mathbf{w} containing all the distinct n th roots of z .

10.* Write a function `ins = inspan(A, b)` to calculate

$$\text{ins} = \begin{cases} 1 & \text{if } \mathbf{b} \in \text{col}(A) \\ 0 & \text{otherwise} \end{cases}$$

11.* Write a function `stab = ctstab(a, b, c)` to calculate

$$\text{stab} = \begin{cases} 1 & \text{if the continuous time system } ax''(t) + bx'(t) + cx(t) = 0 \text{ is stable;} \\ 0 & \text{otherwise.} \end{cases}$$

where the system is stable if and only if $|x(t)| \rightarrow 0$ as $t \rightarrow \infty$.

12.* Write a function `stab = dtstab(a, b, c)` to calculate

$$\text{stab} = \begin{cases} 1 & \text{if the discrete time system } ax_{n+1} + bx_n + cx_{n-1} = 0 \text{ is stable;} \\ 0 & \text{otherwise.} \end{cases}$$

where the system is stable if and only if $|x_n| \rightarrow 0$ as $n \rightarrow \infty$.

13. The text file `data.txt` holds a table consisting of two columns.

- Create two column vectors \mathbf{x} and \mathbf{y} holding the numbers in the first and second columns, respectively.
- Fit a cubic polynomial p to the data points (x_i, y_i) .
- Plot, on the same pair of axes, the data points (x_i, y_i) as blue diamonds and the cubic polynomial p as a solid red line.
- Add a command that saves the plot as an encapsulated postscript file called `dataplot.eps`.

14. Plot, over the interval $0 < t < \pi$, the solution of the initial-value problem

$$(1 + y^2) \frac{dy}{dt} = 2 \cos 3t - y \quad \text{for } t > 0, \quad \text{with } y = 1 \text{ at } t = 0.$$

15. Create a surface plot of

$$z = e^{-x^2} \sin y \quad \text{for } -3 \leq x \leq 3 \text{ and } -2\pi \leq y \leq 2\pi.$$

Label the three axes, add the title and use the `print` command to save the plot to a postscript file `mysurf.ps`.

16. Let

$$z = \frac{2x^2 + \frac{1}{2}y^2}{2 + (x-1)^2 + (y+1)^2} \quad \text{for } -3 \leq x \leq 3 \text{ and } -3 \leq y \leq 3.$$

Plot the contour lines $z = c$ for $c = k/6$ where $k \in \{1, 2, 3, 4, 5, 6, 8, 10, 12, 14, 16, 18\}$.

Notes

- MATLAB uses the names of files to search for functions, so the function `dtstab` must be in the file `dtstab.m`.
- Your answers, apart from the functions, will be in files `q1.m`, `q2.m`, etc. Thus you must not use `q1`, `q2` as the names of variables in MATLAB or there will be confusion between the variable and the script file.
- Do not create any directories (folders) in the exam account: just save all files to your home directory. The Matlab editor will do this anyway if you do not alter any of its default settings.
- In each harder question, you will only be asked to submit the file specifying the function, but first you must test that it works correctly.
- Your M-files should print enough output to answer the question, and to verify that and data has been input correctly. Excessive amounts of output will be penalized.
- MATLAB and Linux are case sensitive, so `B` and `b` are different variables!
This also means that if you are asked to submit an answer in a file `q1.m`, but you submit it in the file `Q1.m`, it will **not** be marked!
- It is a good idea to type the MATLAB command

```
>> clear all
```

between attempts at different questions.

However you must **not** use `clear all` inside any of your solution files.

STUDENT-OWNED COMPUTERS FOR MATHEMATICS COURSES

The School of Mathematics and Statistics is committed to providing, through its own laboratories, all the computing facilities which students need for courses taught by the School. No student should feel the need to buy their own computer in order to undertake any Mathematics course. Nevertheless, the following information is provided for the benefit of those who may wish to use their own computer for work associated with Mathematics courses.

UNSW Moodle may be accessed from any computer with internet access; see their help files and pages for technical requirements and how to check whether your web browser is supported. Some courses may also make use of Maple TA for testing. If you use your own computer to access this system, you should have an up to date browser and java plugin.

The School of Mathematics and Statistics provides assistance to students using teaching software in its laboratories. It does not have the resources to advise or assist students in the use of home computers or in communication between home computers and university facilities.

SOME GREEK CHARACTERS

Listed below are the Greek characters most commonly used in mathematics.

Name	Lower case	Upper case		Name	Lower case	Upper case
Alpha	α			Nu	ν	
Beta	β			Xi	ξ	
Gamma	γ	Γ		Pi	π	Π
Delta	δ	Δ		Rho	ρ	
Epsilon	ϵ			Sigma	σ	Σ
Zeta	ζ			Tau	τ	
Eta	η			Phi	φ or ϕ	Φ
Theta	θ	Θ		Chi	χ	
Kappa	κ			Psi	ψ	Ψ
Lambda	λ	Λ		Omega	ω	Ω
Mu	μ					