UNSW SCIENCE

SCHOOL OF MATHS AND STATISTICS

MATH2111

HIGHER SEVERAL VARIABLE CALCULUS

Term 1, 2019

Cricos Provider Code: 00098G

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1 Course summary

Lecturers

Dr Denis Potapov	RC–6111	d.potapov@unsw.edu.au
Dr Jan Zika	RC–4074	j.zika@unsw.edu.au.

Dr Potapov will lecture weeks 1–5 and Dr Zika will lecture weeks 6–10.

Course Authority: Dr Denis Potapov

Credit: 6 Units of Credit (6UOC).

Prerequisites: MATH1231 or MATH1241 or MATH1251 each with a mark of at least 70.

Exclusions: MATH2019, MATH2049, MATH2069, MATH2100, MATH2110, MATH2011, MATH2039, MATH2510, MATH2610

Lectures: The lectures will be held on weeks 1 to 5 and 6 to 10. Please look for the time and location of each class on myUNSW:

Tutorials: The tutorials begin in week 1. Tutorial problem sheets will be posted on Moodle. You should attempt the problems before attending your tutorial.

2 About this course

This 6UOC course is the Higher Version of the core second year mathematics topic, Several Variable Calculus. Either this course or its ordinary level version MATH2011 is required for completion of a mathematics major. It also forms a compulsory or recommended component of several other programs. MATH2111 is highly recommended for students intending to proceed to Honours.

2.1 Course Aims

The aim of this course is to deepen your understanding of the ideas and techniques of integral and differential calculus for functions of *several* variables. These ideas and techniques are crucial to mechanics, dynamics, electromagnetism, fluid flow and many other areas of pure and applied mathematics. The course combines and extends ideas from one variable calculus and linear algebra to establish the calculus of vector-valued functions: from differentiation through multiple integration to integration over curves and surfaces and the classical Stokes' and Divergence Theorems. The emphasis is on understanding fundamental concepts, developing spatial understanding and acquiring the ability to solve concrete problems.

2.2 Higher or Ordinary

Formally, entry to MATH2111 requires a mark of 70 in first year. Past experience indicates that students who have not achieved this grade struggle with the course. MATH2111 contains a significant amount of extra, theoretical, material compared to MATH2011. Apart from the extra understanding that this brings, the reward for this is that the examination marks are scaled to make sure that the grades are comparable. Many more Distinctions and High Distinctions are awarded in MATH2111 than in MATH2011. The pass rate in MATH2111 is traditionally very high (as it should be with the quality of students in the course). Gaining a high mark in MATH2111 requires that a student understands a reasonable amount of this extra material. Passing requires (as it does in MATH2011) that the student can at least do the most important computational parts of the course.

2.3 Student Learning Outcomes

Students taking this course will

- develop an understanding of the main ideas of calculus in higher dimensions,
- develop proficiency in performing computations arising in higher dimensional calculus,
- become acquainted with the central concepts of mathematical analysis, and of classical applied mathematics that will be used in later years.

Through regularly attending lectures and applying themselves in tutorial exercises, students will develop competency in mathematical presentation, written and verbal skills.

2.4 Relation to graduate attributes

The above outcomes are related to the development of the Science Faculty Graduate Attributes, in particular: 1. **Research, inquiry and analytical thinking abilities**, 4. **Communication**.

2.5 Teaching strategies underpinning the course

New ideas and skills are introduced and demonstrated in lectures, then students develop these skills by applying them to specific tasks in tutorials and assessments.

2.6 Rationale for learning and teaching strategies

We believe that effective learning is best supported by a climate of enquiry, in which students are actively engaged in the learning process. To ensure effective learning, students should participate in class as outlined below.

We believe that effective learning is achieved when students attend all classes, have prepared effectively for classes by reading through previous lecture notes, in the case of lectures, and, in the case of tutorials, by having made a serious attempt at doing for themselves the tutorial problems prior to the tutorials.

Furthermore, lectures should be viewed by the student as an opportunity to learn, rather than just copy down lecture notes. Effective learning is achieved when students have a genuine interest in the subject and make a serious effort to master the basic material.

The art of logically setting out mathematics is best learned by watching an expert and paying particular attention to detail. This skill is best learned by regularly attending classes.

3 Assessment

UNSW assesses students under a standards based assessment policy. For how this policy is applied in the School of Mathematics and Statistics see

http://www.maths.unsw.edu.au/currentstudents/assessment-policies

3.1 Assessment components

All students are required to complete the following assessment tasks

Class Test 1	week 3	10%
Class Test 2	week 6	20%
Class Test 3	week 10	20%
Final Examination	Exam period	50%

3.2 Class Tests

The class tests are designed to give you a chance to assess your mastery of the course material, including both the theoretical and computational aspects of the course.

Each of the class tests will last 40 minutes. They will **NOT** take place in tutorials. They will be held in the lecture theatre.

Class Test 1 covers the topics of lectures of **weeks 1–2**; Class Test 2, covers the topics of lectures of **weeks 3–5**; and Class Test 3, covers the topics of lectures of **weeks 6–9**.

You must bring your **STUDENT ID** card to each test.

Normal exam conditions apply in tests. In particular, you must not bring any kind of written material into the test and you must not try to get assistance from or give assistance to any other person.

You will need to provide your own writing paper for the test.

You should **keep all marked tests** until the end of term in case an error has been made in recording the marks. Your marks will be available online, and you should check these well before the end of term.

If **illness** (or some other circumstance beyond your control) affects your attendance at or performance in a **class test** you will need to apply for special consideration. Further details can be found if you follow the link:

https://www.maths.unsw.edu.au/currentstudents/special-consideration-illness-misadventure

3.3 Final exam

The final exam is the major assessment task. Its purpose is to determine the level of student mastery of both the theoretical and computational course material. The duration of the final exam will be two hours.

If your attendance at, or performance in, the final exam is affected by circumstances beyond your control, you may be able to apply for special consideration. See the "Administrative Matters" section for further information. You should read this information NOW so that you are aware of the rules and procedures for additional assessment.

3.4 Calculator Information

During Class Tests and Examination, you are allowed to use calculators. Students must supply their own calculator. Only calculators on the UNSW list of approved calculators may be used. This list is similar to the list of calculators approved for HSC examinations.

BEFORE the exam period calculators must be given a UNSW "approved calculator" sticker, obtainable from the School of Mathematics and Statistics Office, and other student or Faculty centres.

The UNSW list of calculators approved for use in end of term exams is available at

https://my.unsw.edu.au/student/academiclife/assessment/examinations/Calculator.html

4 Resources for students

4.1 Moodle

This course will use Moodle. Use your zID and zPass to log in to Moodle at the following URL.

http://moodle.telt.unsw.edu.au

4.2 Texts

Vector Calculus, Marsden and Tromba, 6e: The text for the course is *Vector Calculus* (Sixth Edition, 2012) by Marsden and Tromba and published by W. H. Freeman and Company, New York. In particular, there is a webpage to complement the book where you will find further resources,

http://bcs.whfreeman.com/marsdenvc6e/

Further reading: For the more abstract material you should consult more widely. Recommended texts are

- Morgan, F., *Real Analysis*, American Mathematical Society, 2005, P515/91.
- Williamson, Crowell and Trotter, *Calculus of Vector Functions*, P517.5/23.

The first year text *Calculus* by Salas, Hille and Etgen published by Wiley (any recent edition) has material relevant to much of this course.

A useful text for the writing assignment is

- Vivaldi, F., *Mathematical Writing*, Springer 2014, HUC 510/751.

Problem sets: The tutorial exercises will be provided on Moodle.

Sample exams: Sample exams will be provided on Moodle closer to Exam period.

5 Syllabus

The references shown against each topic are to the textbook by Marsden and Tromba [MT] and the two addition texts listed above, Morgan [M] and Williamson, Crowell and Trotter [WCT].

These references are not a definition of what is covered in this course — they are just a guide to finding relevant material. Some parts of the course are not covered in the references and many parts of the references (even in the sections mentioned below) are not included in this course.

1. Multivariable Calculus (Weeks 1-5; D. Potapov)
    - Curves and Surfaces in $\mathbb{R}^n$ [MT, Sections 2.4, 7.3]
        - Week 1 Lecture 1
            * Curves in $\mathbb{R}^n$.
        - Week 1 Lecture 2
            * Surfaces in $\mathbb{R}^n$.
    - Introduction to Analysis and Topology in $\mathbb{R}^n$ [Web notes; M, Parts I and II; MT, Section 2.2]
        - Week 1 Lecture 3
            * Topology of $\mathbb{R}^n$.
        - Week 2 Lecture 1
            * Limits and continuity for functions $\mathbb{R}^m$ to $\mathbb{R}^n$.
            * Compact and connected sets

    - Week 2 Lecture 2
        - Images of compact and connected sets

- Differential Calculus [MT, Sections 2.3, 2.5, 2.6, 3.2–3.5]
    - Week 2 Lecture 3
        - Partial derivatives.
    - Week 3 Lecture 1
        - Differentiability.
        - The Chain Rule
    - Week 3 Lecture 2
        - *Scalar fields* Directional derivatives and the gradient
    - Week 3 Lecture 3
        - Normals and tangents
        - Taylor's Theorem, higher order differentials
    - Week 4 Lecture 1
        - Maxima and minima, Lagrange multipliers
        - *Solvability of systems of non-linear equations*
    - Week 4 Lecture 2
        - The Inverse and Implicit Function Theorems
- Integral Calculus [MT, Sections 5.1–5.5, 6.2]
    - Week 4 Lecture 3
        - Double and triple integrals
        - Iterated integrals and Fubini's Theorem
    - Week 5 Lecture 1
        - Differentiation under the integral sign
        - Change of variables
    - Week 5 Lecture 2
        - Integrals in polar, cylindrical and spherical coordinates

2. Vector Calculus (Weeks 6–9, J. Zika)

    - Prelude
        - Week 5 Lecture 3
            - Scalar and vector fields
            - Dot and cross products
    - Path Integrals [MT, Sections 7.1–7.2]
        - Week 7 Lecture 1

    * Path integral of a scalar field
    * Path integral of a vector field
    * Work and energy
  - Week 7 Lecture 2
    * Properties of path integrals
    * Surface integrals
- Surface Integrals [MT, Sections 7.3–7.7]
  - Week 7 Lecture 3
    * Parametrized surfaces
    * Surface area
    * Integrals of scalar functions over surfaces
  - Week 8 Lecture 1
    * Surface integrals of vector fields
    * Applications.
- Vector Differential Operators [MT, Section 4.4]
  - Week 8 Lecture 2
    * Gradient
    * Curl
    * Divergence
  - Week 8 Lecture 3
    * Two key identities
    * Translating and rotating the frame of reference
- Vector Integral Theorems [MT, Sections 8.1–8.5]
  - Week 9 Lecture 1
    * Two concepts of boundary
    * Green's theorem
    * Stokes's theorem
  - Week 9 Lecture 2
    * Divergence theorem
    * Applications of the integral theorems
    * Differential forms (trailer)
- Orthogonal Curvilinear coordinates [MT, Sections 8.1–8.5]
  - Week 9 Lecture 3
    * General curvilinear coordinates
    * Path, surface and volume integrals

    * Gradient, curl and divergence

3. Fourier Series (week 10; J. Zika)

    - WCT, Sections 5.5, 5.6, 5.8, Appendix 4; M, Sections 17, 22
        - Week 10 Lecture 1
            * Introduction
            * Periodic functions
            * Trigonometric polynomials
        - Week 10 Lecture 2
            * Pointwise vs uniform convergence
            * Trigonometric series
        - Week 10 Lecture 3
            * Piecewise continuous functions
            * Fourier polynomials
            * Fourier series