# UNSW SCIENCE

# SCHOOL OF MATHS AND STATISTICS

# MATH2221

# HIGHER DIFFERENTIAL EQUATIONS

## Term 2, 2019

Cricos Provider Code: 00098G

# MATH2221 – Course Outline

## Information about the course

**Course Authority:** Dr Jan Zika RC-4074.

**Consultation:** See the course home page.

**Credit:** This course counts for 6 Units of Credit (6UOC).

**Prerequisites:** The formal prerequisite is a mark of at least 70 in MATH1231 or MATH1241 or MATH1251.

**Exclusions:** MATH2018, MATH2019, MATH2029, MATH2059, MATH2120 and MATH2130.

**Lectures:** For weeks 1–5 and 7–10 lectures take place at the following times and places.

| | Day | Time | Room |
|---|---|---|---|
| Lecture 1 | Wednesday | 9am-11am | Electrical Engineering G22 |
| Lecture 2 | Thursday | 2pm-4pm | Electrical Engineering G22 |

**Tutorials:** For weeks 1–10 you will attend one tutorial per week. The key to successfully completing this course is to work through the tutorial exercises. Only **some** of these exercises will be solved in the tutorials: you should work through as many of the remaining exercises as you can in your own time.

**TELT:** All course materials will be available on the course home page, which is accessed by logging on to

`http://moodle.telt.unsw.edu.au/my/`.

You should check regularly for new materials.

## Course aims

This course aims to build on your previous study of ordinary differential equations (ODEs) as part of first year calculus. We begin by studying initial-value problems for second and higher-order linear ODEs. Next is an overview of first-order systems of ODEs, touching on a range of topics that are treated at greater depth in our third-year courses. We then return to the topic of linear second-order ODEs, but consider boundary-value problems, as well as a first look at separation of variables for partial differential equations (PDEs). The remainder of the course treats eigenproblems for ordinary and partial differential operators, and their use for solving initial boundary-value problems for PDEs using Cartesian or polar coordinates.

Although the main focus of the course is on analytical methods of solution, we also discuss a variety of applications that give rise to differential equation models.

# Relation to other mathematics courses

MATH2221 is a core second-year course, and will prepare you for a range of later-year courses offered by the School of Mathematics and Statistics that treat topics in the theory and applications of differential equations. In particular, MATH3121 Mathematical Methods and Partial Differential Equations, MATH3201 Dynamical Systems and Chaos and MATH3261 Fluids, Oceans and Climate are natural sequels to MATH2221.

# Student Learning Outcomes

By the end of this course students will be able to:

- Characterise a wide range of ordinary and partial differential equations and solution methods appropriate to them;
- Determine whether solutions to a range of such problems exist and are unique;
- Apply a range of methods to find solutions to differential equations;
- Exploit the concept of linear superposition to solve a range of single and multidimensional initial-boundary value problems;

### Relation to graduate attributes

The above outcomes are related to the development of the Science Faculty Graduate Attributes, in particular: 1.**Research, inquiry and analytical thinking abilities**, 4. **Communication**, 6. **Information literacy**

# Teaching strategies underpinning the course

New ideas and skills are introduced and demonstrated in lectures. Students will develop these skills by applying them to specific tasks in tutorials and tests. They will further these skills by applying them in the written assignments and reflect on them through peer review.

### Rationale for learning and teaching strategies

We believe that effective learning is best supported by a climate of enquiry, in which students are actively engaged in the learning process. To ensure effective learning, students should participate in class as outlined below.

We believe that effective learning is achieved when students attend all classes, have prepared effectively for classes by reading through previous lecture notes, in the case of lectures, and, in the case of tutorials, by having made a serious attempt at doing for themselves the tutorial problems prior to the tutorials.

Furthermore, lectures should be viewed by the student as an opportunity to learn, rather than to copy down lecture notes.

Effective learning is achieved when students have a genuine interest in the subject and make a serious effort to master the basic material.

The art of logically setting out mathematics is best learned by watching then making attempts oneself. This skill is best learned by regularly attending classes, attempting problems between lectures and tutorials , reviewing ones work and seeking feedback from fellow students and tutors.

# Assessment

The course has four assessment tasks:

| Task | Time | Contribution |
|---|---|---|
| Assignment 1 | Week 3-4 | 10% |
| Class Test | 17th July (Week 7) | 20% |
| Assignment 2 | Week 8-9 | 10% |
| Final Exam | TBC | 60% |

## Assignments

There will be two assignments in this course with a written component and a peer assessment component in each.

The aim of the written component is to develop your ability to present mathematics in a professional way, paying attention to neatness, grammar, clarity of argument, use of notation and so forth. It will also help you to develop a deeper understanding about the material taught in class and its applications.

The peer assessment part of the assignment will be administered using Moodle. The peer review part will develop your ability to critically scrutinise work by others, identify a range of methods of presenting mathematical argument and to provide feedback to your peers.

Assessment of the assignments will be 10% each. For each assignment the peer review aspect will contribute 4% and the mark for the assignment itself will constitute 6%.

Further details of what is expected, including submission dates for the peer review and final submissions, will be announced in lectures and on Moodle.

## Test

The class test is designed to give you a chance to assess your mastery of the course material, including both theoretical and computational aspects of the course.

Announcements will be made in lectures about the topics that will be covered in the test.

You must bring your **STUDENT ID** card to the test.

Normal exam conditions apply in the test. In particular, you must not bring any kind of written material into the test and you must not try to get assistance from or give assistance to any other person.

You will need to provide your own writing paper for the test.

You will **NOT** be allowed to use a calculator in class tests.

You should **keep all marked tests** until the end of semester in case an error has been made in recording the marks. Your marks will be available online, and you should check these well before the end of semester.

## Final exam

The final exam is the major assessment task. It will last for 2 hours. Its purpose is to determine the level of student mastery of both the theoretical and computational course material. The duration of the final exam will be two hours.

### Special consideration

If **illness** (or some other circumstance beyond your control) affects your ability to complete any of the assessment tasks you will need to apply for special consideration.

Special Consideration web site: `https://student.unsw.edu.au/special-consideration`

For final exams with special consideration granted, the Exams Unit will email the rescheduled "supplementary exam" date, time and location to your student zID email account directly.

For successful applications for special consideration, please ensure you regularly check your student email account (zID account) for the information on new dates to attend an assessment, or dates for any "supplementary exam" both in Term and Final.

The supplementary exam period/dates for the final exam can be found at this web site: https://student.unsw.edu.au/exam-dates Please ensure you are aware of these dates and that you are available during this time.

### Calculator Information

For end of semester UNSW exams student must supply their own calculator. Only calculators on the UNSW list of approved calculators may be used in the end of semester exams. This list is similar to the list of calculators approved for HSC examinations.

BEFORE the exam period calculators must be given a UNSW "approved calculator" sticker, obtainable from the School of Mathematics and Statistics Office, and other student or Faculty centres.

The UNSW list of calculators approved for use in end of semester exams is available at

`https://my.unsw.edu.au/student/academiclife/assessment/examinations/Calculator.html`

## Additional resources and support

#### Tutorial Exercises

A set of tutorial exercises will be given out. These problems are for YOU to do to enhance mastery of the course.

SOME of the problems will be done in tutorials, but you will learn a lot more if you try to do them before the tutorial.

### Course materials

The outline lecture notes, tutorial problems and any other course materials will be provided via the course home page.

### Reference books

## Reference books

There is no prescribed textbook for the course, but you may find the following texts useful:

- Zill, Dennis G. Differential equations with boundary-value problems. Nelson Education, 2016.
- W.E. Boyce and R.C. DiPrima, Elementary Differential Equations and Boundary Value Problems, Wiley, P515.35/18.
- E. Kreyszig, Advanced Engineering Mathematics, Wiley, P510.2462/5.
- Steven H. Strogatz, Nonlinear Dynamics and Chaos, Addison-Wesley, P531.11/94.
- R.K. Nagle and E.B. Saff, Fundamentals of Differential Equations and Boundary Value Problems - 4th Edition, Addison-Wesley, P517.382/197.

The content of the course will be defined by the lectures and the tutorial problems.

**Problem sets.** The tutorial exercises will be provided as PDFs on Moodle.

**Sample exams.** Moodle will contain a number of sample exams.

## Course Evaluation and Development

The School of Mathematics and Statistics evaluates each course each time it is run. We carefully consider the student responses and their implications for course development. Feedback is very important to us, so please don't leave it to the end of the course to pass on any ideas.

## School Rules and Regulations

Details of the general rules regarding attendance, release of marks, special consideration etc are available via the School of Mathematics and Statistics Web site:

http://www.maths.unsw.edu.au/currentstudents/assessment-policies

In particular, for the additional assessment policies see

http://www.maths.unsw.edu.au/currentstudents/additional-assessment

The dates for the additional assessment exams are available from the online Student Noticeboard.

## Course schedule

The following list is an *approximate* schedule of topics covered in this course. The course content is ultimately defined by the material covered in the lectures and the schedule is likely to change throughout the term.

### Linear ODEs

- Week 1 Lecture 1
  - Linear differential operators
  - Differential operators with constant coefficients
- Week 1 Lecture 2
  - Wronskians and linear independence
  - Methods for inhomogeneous equations
- Week 2 Lecture 1
  - Solution via power series
  - Singular ODEs
- Week 2 Lecture 2
  - Bessel and Legendre equations

### Dynamical systems: an overview

- Week 3 Lecture 1
  - Examples and terminology
  - Existence and uniqueness

- Week 3 Lecture 2
  - Linear dynamical systems
  - Stability
- Week 4 Lecture 1
  - Classification of 2D systems
  - Final remarks on nonlinear DEs

### Initial-boundary value problems in 1D

- Week 4 Lecture 2
  - Two-point boundary-value problems
  - Existence and uniqueness
- Week 5 Lecture 1
  - Inner products and norms of functions
  - Self-adjoint differential operators

### Generalised Fourier series

- Week 5 Lecture 2
  - The vibrating string
  - Complete orthogonal systems
- Week 7 Lecture 1
  - Class test
- Week 7 Lecture 2
  - Complete orthogonal systems continued
- Week 8 Lecture 1
  - Sturm–Liouville problems
- Week 8 Lecture 2
  - Fourier–Bessel series
  - Schrödinger equation

## Initial-boundary value problems in 2D

- Week 9 Lecture 1
  - Elliptic differential operators
  - Green identities and boundary value problems
- Week 9 Lecture 2
  - Elliptic eigenproblems
- Week 10 Lecture 1
  - Wave and diffusion equations
- Week 10 Lecture 2
  - Revision