Faculty of Science
School of Mathematics and Statistics

MATH2701

Abstract Algebra and Fundamental Analysis

Semester 2 2013
Course information
MATH2701 is a 6 UOC second year course intended for students in the Advanced Mathematics program, especially those considering majoring in Pure Mathematics. It is designed to bridge the gap between the second year core courses and the greater level of abstraction required in higher level III Mathematics courses.

- Prerequisites: MATH1231 or MATH1241 or MATH1251 with at least a CR, and enrolment in an Advanced Mathematics or Advanced Science program. (Other students may be allowed entry to the course if space permits. Approval may be sought from the course lecturers.)

Course structure
There will be 3 lectures and one tutorial per week.

Course staff
Prof. Jie Du, RC-4113, phone 9385-7087, email J.Du@unsw.edu.au
A/Prof. Ian Doust, RC-6113, phone 9385-7097, email i.doust@unsw.edu.au
  o Office consultation: by appointment.

Location and Times
- Lectures
  o Tuesday 11 – 12 EE-G25
  o Wednesday 12 – 01 ME-405
  o Thursday 09 – 10 ME-405
- Tutorials
  o Thursday 10 – 11 OMB-145A

Course description
Mathematics went through quite a revolution around the turn of the 20th century. In particular, an axiomatic approach infiltrated the mathematical paradigm, both as a tool to ensure mathematical rigour and to abstract common principles working in a variety of different settings.

First year mathematics emphasizes computation over abstraction and rigour. Later year courses (and Pure Mathematics in general) reverse this, so students need to learn some new skills and some new ways of thinking about mathematical objects.

This course is designed to help you develop the ability to write rigorous mathematical proofs in a setting where the level of abstraction is still quite modest. As such it will serve as an excellent preparation for the third year Pure Mathematics courses.

The course consists of two halves, algebra and analysis, each taught for 6 weeks.

In the algebra half, we will investigate various transformations on the plane and projective plane. We will first study several types of transformations such as translations, reflections, rotations etc. in terms of groups. We will then look at symmetries, i.e. transformations of geometric figures that preserve some property (such as distance or angles between lines), and projective geometry. Projective transformations can change a conic section of one type to another, e.g. an ellipse to a hyperbola.

Most of the calculus you have seen involves equalities. Mathematical analysis however, is largely about inequalities, about suitably bounding quantities that cannot be calculated precisely. Many nice examples come from geometry and we will frequently use these to motivate our discussion in the first part of the analysis section. In the latter part we will look more closely at some aspects of the real numbers, such as how well one can approximate $\pi$ by a rational $\frac{p}{q}$ (in terms of how large $q$ is).
Preparation:
You need to know some linear algebra, up to the level of MATH1241 or MATH1251 and some basic naive set theory as you might pick up in those courses and, ideally a discrete maths course. If you haven't done discrete maths, don't worry, the only 'non-trivial' bits of set theory you really need are the notions of products of sets and equivalence relations.

Expected Learning Outcomes
Students are expected to:
- learn how to write mathematics and mathematical proofs.
- understand the basic theory of transformation groups.
- gain some knowledge of groups, especially symmetry groups.
- be able to estimate analytical quantities and to prove the estimates.
- Know some of the main results on inequalities that are used in analysis.

Relation to graduate attributes
The above outcomes are related to the development of the Science Faculty Graduate Attributes, in particular: Research, inquiry and analytical thinking abilities; and, Communication.

Course Evaluation and Development
The School of Mathematics evaluates each course each time it is run. Feedback on the course is gathered, using among other means, UNSW's Course and Teaching Evaluation and Improvement (CATEI) Process. As this is the first time this course is being run, student feedback will be vital in enabling us to make improvements for future years.

Assessment
The grade for this course will be determined from a combination of assignments and a final exam.

Assignments:
As a major aim of this course is to teach you to write mathematics, it is important that we can give you feedback on how you are doing. For each half of the course there will be a number of short assignments and as well as a more substantial assignment. The short assignments will be designed to help you develop your ability to write mathematical proofs. The main assignments will allow you spend time attempting problems that are less straightforward, and then to polish your solutions. The assignments will be handed out regularly throughout the course, with the major assignments due at the end of each part.

Final exam
The final exam is the major assessment task. Its purpose is to determine the level of student mastery of the course material. The duration of the final exam will be two hours.

Your raw course mark will be calculated with the following weightings:

- Short assignments: 30% (at most 5% per assignment)
- Main assignments: 20% (10% each)
- Final exam: 50%

The final marks may be scaled according to policies of the university and the School of Mathematics and Statistics. If your attendance at, or performance in, the final exam is affected by circumstances beyond your control, you may be able to apply for special consideration. See the "Administrative Matters" section for further information. You should read this information NOW so that you are aware of the rules and procedures for additional assessment.
Teaching strategies underpinning the course
New ideas and skills are introduced and demonstrated in lectures, then students develop these skills by applying them to specific tasks. Students should attend all classes, and work through the tutorial problems prior to the tutorial. The assignments and the feedback provided to these form a central part of the learning process in this course. The course is structured to create a climate of enquiry in which students are actively engaged in the learning process.
Syllabus
The course will include material from the following. The course content is ultimately defined by the material covered in lectures.

Part 1: Algebra
Rigid transformations of the plane and examples: permutations of $\mathbb{R}^2$, linear isomorphism, collineations, translations, halfturns; Groups of transformations: definitions, examples, subgroups, cyclic groups; the translation group, abelian groups; the group generated by half-turns; the group generated by reflections, the groups of isometries and rotations; the symmetry group of a plane figure; the dihedral group; finite groups of plane isometries; Affine geometry: the group of affine transformations; Projective geometry: the extended Euclidean plane, points and lines at infinity, principle of duality, modelling the real projective plane by rays in space, homogeneous coordinates, lines and conics in the real projective plane; the group of projective transformations.

Part 2: Analysis
Some motivating questions; asymptotics, “big O and little o” notation; standard inequalities in analysis; norms and metrics; convex bodies and dual norms; isoperimetric inequalities; packing problems; norms on the rational numbers and Ostrowski’s theorem; construction of the real numbers; approximation of irrational numbers in base $b$; $p$-adic numbers.

References
The lectures, tutorials and problem sheets will cover all the material that you need to know, but nevertheless, you will probably find it handy to supplement your studies by looking at texts such as those below.


Library
- The library has a mathematics subject guide on the web which is a good starting point for mathematical information. They can be found at http://info.library.unsw.edu.au/ and http://info.library.unsw.edu.au/psl/guides/math/mathkey.html
Administrative Matters

Students should be aware of a number of standard School and University policies which apply for this course. Most information is available from the School of Mathematics and Statistics Student Services site:  
http://www.maths.unsw.edu.au/currentstudents/student-services

Additional Assessment

- The School of Mathematics has a strict policy on additional assessment. It can be found at http://www.maths.unsw.edu.au/currentstudents/additional-assessment. Note in particular the special arrangements for students in second year mathematics courses whose final mark is in the range 40-49.

Plagiarism and academic integrity

- Plagiarism is the presentation of thoughts or work of another as one’s own. Issues you must be aware of regarding plagiarism and the university’s policies on academic integrity can be found at http://www.lc.unsw.edu.au/plagiarism.

Academic Misconduct

- The University of New South Wales has rules relating to Academic Misconduct. They can be found at http://www.maths.unsw.edu.au/currentstudents/policy-academic-misconduct.

Rules for the Conduct of Examinations

- The University of New South Wales has rules for the conduct of examinations. They can be found at http://www.maths.unsw.edu.au/currentstudents/rules-exams.

Occupational Health and Safety


Equity and Diversity

- Those students who have a disability that requires some adjustment in their teaching or learning environment are encouraged to discuss their study needs with the course convener prior to, or at the commencement of, their course, or with the Equity Officer (Disability) in the Equity and Diversity Unit (9385 4734 or http://www.studentequity.unsw.edu.au). Issues to be discussed may include access to materials, signers or note-takers, the provision of services and additional exam and assessment arrangements. Early notification is essential to enable us to best meet your needs.