FACULTY OF SCIENCE

SCHOOL OF MATHEMATICS & STATISTICS

MATH3161/MATH5165

OPTIMIZATION

COURSE OUTLINE

SEMESTER 1, 2018

MATH3161/MATH5165 Optimization

Lecturer

- Prof. Jeya Jeyakumar, RC-2073, Ph: 9385-7046, Email: v.jeyakumar@unsw.edu.au (Course Authority)

Course details

- **Units of Credit** 6
- **Prerequisites** 12 units of credit in Level 2 Mathematics courses including MATH2011 or MATH2111 or MATH2510, and MATH2501 or MATH2601, or both MATH2019(DN) and MATH2089, or both MATH2069(CR) and MATH2099.
- **Exclusions** MATH3181

Lectures

The times and locations of the lectures are

Day	Time	Location
Monday	14:00	CLB 5
Monday	15:00	CLB 5
Wednesday	9:00	MATHEWS-C

The lectures will be common to both MATH3161 and MATH5165 students. Students in the graduate version (MATH5165) are expected to display much more independence, working through all tutorial problems in their own.

Tutorials

There will be one tutorial per week. Tutorials start in Week 2.

Consultation

Please use e-mail to arrange an appointment with your lecturer.

Course Web Site

The School of Mathematics and Statistics uses the Learning Management System called Moodle. To log into Moodle, use your zID and zPass at the following URL:

`http://moodle.telt.unsw.edu.au`

The MATH3161/MATH5165 course website will be available through UNSW Moodle. Here you will find announcements, general information, notes, lecture slides, problem sheets and lecture supplements.

Course Aims

The concept of optimization, finding the "best" way to do something, arises across all branches of mathematics and in application areas ranging from biology and engineering to business and finance. The purpose of this course is to provide an introduction to the theory of multi-variable optimization and optimal control, and to provide students with the skills to formulate, solve and analyze solutions to certain multi-variable optimization problems and infinite dimensional optimal control problems.

Relation to other mathematics courses
This course has a major focus on nonlinear continuous optimization problems, as distinct from linear programming problems and discrete optimization problems.

Teaching Strategies Underpinning the Course

New concepts and skills are first introduced and demonstrated in lectures, then students develop theses skills by applying them to specific tasks in tutorials and assessments.

Rationale for learning and teaching strategies
We believe that effective learning is best supported by a climate of inquiry, in which students are actively engaged in the learning process. Hence, this course is structured with a strong emphasis on problem-solving tasks in lectures, tutorials and in assessment tasks, and students are expected to devote the majority of their class and study time to the solving of such tasks.
To ensure effective learning, students should participate in class as outlined below.

We believe that effective learning is achieved when students attend all classes, have prepared effectively for classes by reading through previous lecture notes, in the case of lectures, and, in the case of tutorials, by having made a serious attempt at doing for themselves the tutorial problems prior to the tutorials.

Furthermore, lectures should be viewed by the student as an opportunity to learn, rather than just copy down lecture notes.

Effective learning is achieved when students have a genuine interest in the subject and make a serious effort to master the basic material.

The art of logically setting out mathematics is best learned by watching an expert and paying particular attention to detail. This skill is best learned by regularly attending classes.

Expected Student Learning Outcomes

Students taking this course will develop an appreciation of the basic problems of optimization and skills to solve optimization problems. Computing skills are developed and practised in attempting assessment tasks.
By the end of the course students should be able to formulate, solve and analyze solutions to certain optimization problems. The ability to solve optimization problems via analytical, numerical and computational methods will be paramount.
Through regularly attending lectures, attempting assessment tasks, and applying themselves in tutorial exercises, students will develop competency in mathematical presentation, written and verbal skills.
Relation to graduate attributes: The above outcomes are related to the development of the Science Faculty Graduate Attributes, in particular: 1.**Research, inquiry and analytical thinking abilities**, 4. **Communication**, 6. **Information literacy**

Assessment

UNSW assesses students under a standards based assessment policy. For how this policy is applied in the School of Mathematics and Statistics see
http://www.maths.unsw.edu.au/currentstudents/assessment-policies

There will be two (2) class tests, one (1) assignment and a final exam.

Task	Date	Weighting	Duration
Class Test 1	Wednesday, Week 7	16%	50 mins
Class Test 2	Wednesday, Week 11	16%	50 mins
Assignment	Week 12	8%	——
Final Exam	June	60%	2 hrs
Total		100%	

- **Class Tests and Quizzes:** There will be two class tests counting 32% of the total assessment. Details of the class tests will be announced during lectures. There will also be two quizzes counting 0% of the total assessment.
- The Class Tests are held in place of the Wednesday lecture. Location for the class tests will be announced in lectures – it is your responsibility to get this information. The quizzes will be given during lectures.
  - **Rationale.** The Tests and quizzes will give students feedback on their progress and mastery of the material.
  - There will be short answer questions in which correct answers are sought and there will be some longer questions requiring clear and logical presentation of correct solutions as well as some simple proofs and verbal explanations.
- **Final Exam:** The final exam, covering *everything* in the course, counts for 60% of the total assessment. Further details about the final examination will be available in class closer to the time.
  - **Duration:** Two hours.
  - **Rationale:** The final examination will assess student mastery of the material covered in lectures, tutorials, problems sheets, and any distributed material.
- **Starred Materials:** Problem sheets, class tests and the June exam may have starred questions indicating harder material. Grades of Pass and Credit can be gained by satisfactory performance on unstarred questions. Grades of Distinction and High Distinction will require satisfactory performance on all questions.
  - Students in the graduate version (MATH5165) are expected to show satisfactory performance on starred questions.
- **Assignment** It is planned to have an assignment on solution methods of optimization problems. The assignment may involve modelling practical problems and writing a short report. The MATLAB software package may also be used for implementing numerical optimization methods to solve practical optimization problems. You are not required to know MATLAB before this course. On-line help in MATLAB is available by typing `help` or `help subject`. It is not assumed that you have done computing subjects.
  - Students in the graduate version (MATH5165) are expected to complete additional work in the assignment.
  - **Rationale:** Assignments will give an opportunity for students to try their hand at more difficult problems requiring more than one line of argument and also introduce them to aspects of the subject which are not explicitly covered in lectures. The assignment will also require a student to draw together several topics in the course.

- Assignments must be YOUR OWN WORK, or severe penalties will be incurred.
  You should consult the University web page on plagiarism
  `https://student.unsw.edu.au/plagiarism`

Additional Resources and Support

- **Calculators**

  You may bring your own UNSW approved Scientific Calculator to the class tests and final exam. Calculators will not be provided for you.

- **Text and Reference Books**

  There is NO textbook which covers all aspects of this course. General reference books are detailed in the last section.

- **Tutorial Exercises**

  Problem sheets for tutorials will be provided via UNSW Moodle. These problems are for you to do to enhance mastery of the course.

  SOME of the problems will be done in tutorials, but you will learn a lot more if you try to do them before the tutorial.

- **Lecture Notes**

  A set of skeleton notes and summary sheets containing only definitions, theorems and proofs will be provided for SOME components of the course on UNSW Moodle.

- **Library**

  The library has a mathematics subject guide on the web which is a good starting point for mathematical information. See
  `http://subjectguides.library.unsw.edu.au/`

Course Evaluation and Development

The School of Mathematics evaluates each course each time it is run. Feedback on the course is gathered, using among other means, UNSW's Course and Teaching Evaluation and Improvement (CATEI) Process. Student feedback is taken seriously, and continual improvements are made to the course based in part on such feedback.

Administrative Matters

- **Special Consideration**

  Special consideration for class tests or the assignment or the final exam will *only* be granted on medical or compassionate grounds and *must be documented*. If you miss a class test or the assignment or the June exam, then you must follow the School of Mathematics and Statistics standard procedures set out in the 'Important Information for Students' document.

  See
  `http://www.maths.unsw.edu.au/currentstudents/student-services`

  Visit website for further information on how to Apply for Special Consideration:
  `https://student.unsw.edu.au/special-consideration`

- **Additional Assessment**

  The School of Mathematics has a strict policy on additional assessment. The School policy on special considerations for second and later years subjects will be strictly adhered to. Students must read and understand the School of Mathematics and Statistics Policies as contained in the 'Information for Students' document. See

  https://www.maths.unsw.edu.au/currentstudents/additional-assessment
  http://www.maths.unsw.edu.au/currentstudents/help-students-undergraduate
  http://www.maths.unsw.edu.au/currentstudents/help-students-postgraduate

- **Academic Misconduct**

  The University of New South Wales has rules relating to Academic Misconduct. See
  https://student.unsw.edu.au/conduct

- **Rules for the conduct of examinations**

  The University of New South Wales has rules for the conduct of examinations. See
  http://www.maths.unsw.edu.au/currentstudents/assessment-policies

Detailed Course Descriptions

- **Overview:** Optimization is an area of mathematics that directly deals with the problem of making the best possible choice from a set of feasible choices. It seeks to understand how we achieve the best possible choice and how we can use this knowledge to improve management and technical decisions in science, engineering and commerce. Thinking in terms of choices is common in our cognitive culture and searching for the best possible choice is a basic human desire.Thus models of optimization arise everyday as management and technical decisions in many areas of human activity.

  Problems of engineering design (such as the design of electronic circuits subject to a tolerancing and tuning provision), information technology (such as the extraction of meaningful information from large databases and the classification of data), financial decision making and investment planning (such as the selection of optimal investment portfolios), and transportation management and so on arise in the form of a multi-variable optimization problem or an optimal control problem.

  Optimization has its foundation in the development of calculus by Newton and Leibniz in the 17^{th} century. The solution of large multi-variable optimization problems using computers started with the work of Dantzig in the late 1940s and 1950s on the simplex method for linear programming. Now, multi-variable optimization problems with hundreds of variables can be solved routinely.

- **Introduction:** What is an optimization problem? Areas of applications of optimization. Modelling of practical optimization problems.

- **Multi-variable optimization**

  - **Mathematical background:** Formulation of multi-variable optimization problems; Structure of optimization problems: objective functions and constraints. Mathematical background: multi-variable calculus and linear algebra; (strict) local and (strict) global minimizers and maximizers.
  - **Convexity of sets and functions:** convex sets, convex and concave functions; epigraphs of convex functions; global extrema and uniqueness of solutions.

- **Optimality principles:** First and second order conditions for unconstrained problems; Lagrange multiplier conditions for equality constrained problems; Kuhn-Tucker conditions for both equality and inequality constrained problems.
- **Convex optimization:** Kuhn-Tucker sufficient optimality conditions; duality.
- **Numerical Methods for Unconstrained Problems:** Steepest descent method, Newton's method, Conjugate gradient methods.
- **Numerical Methods for Constrained Problems:** Penalty Methods.

- **Optimal Control**
  - What is an optimal control problem? Areas of applications of optimal control. Mathematical background: ordinary differential equations and systems of linear differential equations.
  - **The Pontryagin maximum principle:** Autonomous systems, autonomous control problems; unbounded controls; non-autonomous control problems.

Reference Books

The general references on optimization are listed below. The standard of the references is somewhat higher than is required in MATH3161/MATH5165.

Optimization References: General references on multi-variable optimization include [1, 4, 10] and on optimal control include [8, 11]

Linear Algebra and Differential Equations: Solving multi-variable optimization problems requires techniques from linear algebra, whereas solving optimal control problems requires solution methods of differential equations. An elementary treatment of linear algebra can be found in Strang [13], while a reference for differential equations is Zill [14].

Mathematical Software: Solving practical problems typically requires a computer software package like MATLAB [9] (see Pratap [12] for an introduction).

References

[1] A. Beck, Introduction to Nonlinear Optimization – Theory, Algorithms and Applications with MATLAB, MOS-SIAM Series on Optimization. SIAM, 2014.

[2] D. P. Bertsekas Nonlinear programming: Second edition, Athena Scientific, Belmont, MA, 1999.

[3] J. E. Dennis and R. B. Schnabel, *Numerical Methods for Unconstrained Optimization and Nonlinear Equations*, SIAM Publications, Classics in Applied Mathematics, 1996.

[4] R. Fletcher, *Practical Methods of Optimization, 2nd Edition*, John Wiley, 2000.

[5] P. E. Gill, W. Murray, and M. H. Wright, *Practical Optimization*, Academic Press, New York and London, 1981.

[6] G. H. Golub and C. F. Van Loan, *Matrix Computations*, John Hopkins University Press, Baltimore and London, third ed., 1996.

[7] J. B. Hiriart-Urruty and C. Lemarechal, *Convex Analysis and Minimization Algorithms*, Springer-Verlag, Berlin, 1993

[8] L. M. Hocking, *Optimal Control: An Introduction to the Theory with Applications*, Oxford University Press, Oxford, 1991.

[9] MathWorks, *MATLAB & Simulink Student Version R2012A*, Englewood Cliffs, 2012. (See UNSW Bookshop `http://www.bookshop.unsw.edu.au/computing/`. The student edition includes the Optimization toolbox)

[10] J. Nocedal and S. J. Wright, *Numerical optimization*, Springer, (2nd edition) 2006.

[11] E. R. Pinch, *Optimal control and the calculus of variations*, Oxford University Press, Oxford, 1995.

[12] R. Pratap, *Getting started with MATLAB: A Quick Introduction for Scientists and Engineers*, Oxford University Press, 2009.

[13] G. Strang, *Linear Algebra and its Applications*, Harcourt Brace Jovanovich, San Diego, 3 ed., 1988.

[14] D. G. Zill, *Differential equations with boundary-value problems*, Second Edition, PWS-Kent Publishing company, Boston, 1989.