UNSW SYDNEY

UNSW SCIENCE

SCHOOL OF MATHEMATICS & STATISTICS

COURSE OUTLINE

MATH3161/MATH5165

OPTIMIZATION

TERM 1, 2021

MATH3161/MATH5165 Optimization

Lecturer

- Prof. Jeya Jeyakumar, RC-2073, Ph: 9385-7046, Email: v.jeyakumar@unsw.edu.au (Course Authority)

Course details

- **Units of Credit** 6
- **Prerequisites** 12 units of credit in Level 2 Mathematics courses including MATH2011 or MATH2111 or MATH2510, and MATH2501 or MATH2601, or both MATH2019(DN) and MATH2089, or both MATH2069(CR) and MATH2099.
- **Exclusions** MATH3181

Lectures

There will be 4 hours of lectures per week (in week 1–5 and week 7-10), giving a total of 36 hours. During week 6 there will be no lectures. The times of the lectures are

Day	Time	Location
Thursday	11:00	On-line
Thursday	12:00	On-line
Friday	11:00	On-line
Friday	12:00	On-line

The lectures will be common to both MATH3161 and MATH5165 students. Students in the graduate version (MATH5165) are expected to display much more independence, working through all tutorial problems in their own.

Tutorials

There will be one on-line tutorial per week, giving a total of 9 hours of tutorials. Tutorials start in Week 1. During week 6 there will be no tutorials.

Moodle and Course Web Site

The School of Mathematics and Statistics uses the Learning Management System called Moodle. The MATH3161/MATH5165 course website will be available through UNSW Moodle.

Log in to Moodle to find announcements, general information, consultation times, notes, lecture slides, tutorial problems and supplementary materials etc. To log into Moodle, use your zID and zPass at the following URL:

`http://moodle.telt.unsw.edu.au/login/`

Administrative Matters

Please visit the School of Mathematics and Statistics web-site for a range of information on School Policies, Forms and Help for Students.

For information on Courses, please go to "Current Students" and either Undergraduate and/or Postgraduate", Course Homepage" for information on all course offerings,

The "Student Notice Board" can be located by going to the "Current Students" page; Notices are posted regularly for your information here. Please familiarise yourself with the information found in these locations. The School web page is:

`https://www.maths.unsw.edu.au`

If you cannot find the answer to your queries on the web you are welcome to contact the Student Services Office directly by email:

Undergraduate: ug.mathsstats@unsw.edu.au; Postgraduate: pg.mathsstats@unsw.edu.au

By phone: 9385 7011 or 9385 7053

Or in person to the Red Centre building, level 3, rooms 3072 or 3088

Should we need to contact you, we will use your official UNSW email address of in the first instance. It is your responsibility to regularly check your university email account. Please state your student number in all emails.

Course Aims

The concept of optimization, finding the "best" way to do something, arises across all branches of mathematics and in application areas ranging from biology and engineering to finance and medicine. The purpose of this course is to provide an introduction to the theory of multi-variable optimization and optimal control, and to provide students with the skills to formulate, solve and analyze solutions to certain multi-variable optimization problems and infinite dimensional optimal control problems.

Relation to other mathematics courses

This course has a major focus on nonlinear continuous optimization problems, as distinct from linear optimization problems and discrete optimization problems which are covered in MATH3171, Linear and Discrete Optimization Modelling, in Term 3.

Assessment

UNSW assesses students under a standards based assessment policy. For how this policy is applied in the School of Mathematics and Statistics see

http://www.maths.unsw.edu.au/currentstudents/assessment-policies

There will be two (2) class tests, one (1) assignment and a final exam.

Task	Date	Weighting	Duration
On-line Class Test 1	Week 5	15%	40 mins
On-line Class Test 2	Week 9	20%	40 mins
Assignment	Week 10	5%	———
Final Exam	May	60%	2 hrs
Total		100%	

- **Class Tests:** There will be two on-line class tests counting 35% of the total assessment. Details of the class tests will be announced during lectures and will also be available on the course website through UNSW Moodle. There will also be two practice on-line quizzes counting 0% of the total assessment.
- The on-line Class Tests are held in place of the second half of the Friday lecture (12:00noon-1:00pm) in weeks 5 and 9.
  - **Rationale.** The Tests will give students feedback on their progress and mastery of the material.
  - There will be short answer questions in which correct answers are sought and there will be some longer questions requiring clear and logical presentation of correct solutions as well as some simple proofs and verbal explanations.
  - On-line quizzes will give you multiple opportunities to practice the key concepts and techniques, learned in the lectures and tutorials, and provide instant feedback on answers.
- **Final Exam:** The final exam, covering *everything* in the course, counts for 60% of the total assessment. Further details about the final examination will be available in class closer to the time.
  - **Duration:** Two hours.

  - **Rationale:** The final examination will assess student mastery of the material covered in lectures, tutorials, problems sheets, and any supplementary material.

- **Starred Materials:** Problem sheets, class tests and the final exam may have starred questions indicating harder material. Grades of Pass and Credit can be gained by satisfactory performance on unstarred questions. Grades of Distinction and High Distinction will require satisfactory performance on all questions.

  - Students in the graduate version (MATH5165) are expected to show satisfactory performance on starred questions.

- **Assignment:** It is planned to have an assignment on optimization methods to solve practical problems. The assignment may involve modelling practical problems and writing a short report. The MATLAB software package may also be used for implementing numerical optimization methods to solve practical optimization problems. You are not required to know MATLAB before this course. On-line help on MATLAB will be available. It is not assumed that you have done computing subjects.

  - Students in the graduate version (MATH5165) are expected to complete additional work in the assignment.
  - **Rationale:** Assignments will give an opportunity for students to try their hand at more difficult problems requiring more than one line of argument and also introduce them to aspects of the subject which are not explicitly covered in lectures. The assignment will also require a student to draw together several topics in the course.
  - Assignments must be YOUR OWN WORK, or severe penalties will be incurred.
    You should consult the University web page on plagiarism
    `https://student.unsw.edu.au/plagiarism`

Computing lab. The main computing laboratory is Room G012 of the Red Centre. You can get to this lab by entering the building through the main entrance to the School of Mathematics (on the Mezzanine Level) and then going down the stairs to the Ground Level. A second smaller lab is Room M020, on the mezzanine level of the Red Centre.

For more information, including opening hours, see the computing facilities webpage:
`https://www.maths.unsw.edu.au/currentstudents/computing-facilities`
Remember that there will always be unscheduled periods when the computers are not working because of equipment problems and that this is not a valid excuse for not completing tests on time.

Teaching Strategies Underpinning the Course

New concepts and skills are first introduced and demonstrated in lectures, then students develop theses skills by applying them to specific tasks in tutorials and assessments.

Rationale for learning and teaching strategies
We believe that effective learning is best supported by a climate of inquiry, in which students are actively engaged in the learning process. Hence, this course is structured with a strong emphasis on problem-solving tasks in lectures, tutorials and in assessment tasks, and students are expected to devote the majority of their class and study time to the solving of such tasks.

To ensure effective learning, students should participate in class as outlined below.

We believe that effective learning is achieved when students attend all classes, have prepared effectively for classes by reading through previous lecture notes, in the case of lectures, and, in the case of tutorials, by having made a serious attempt at doing for themselves the tutorial problems prior to the tutorials.

Furthermore, lectures should be viewed by the student as an opportunity to learn, rather than just copy down lecture notes.

Effective learning is achieved when students have a genuine interest in the subject and make a serious effort to master the basic material.

The art of logically setting out mathematics is best learned by watching an expert and paying particular attention to detail. This skill is best learned by regularly attending classes and watching supplementary course videos.

Expected Student Learning Outcomes

Students taking this course will develop an appreciation of the basic problems of optimization and skills to solve optimization problems. Computing skills are developed and practised in attempting assessment tasks. By the end of the course students should be able to formulate, solve and analyze solutions to certain optimization problems. The ability to solve optimization problems via analytical, numerical and computational methods will be paramount.
Through regularly attending lectures, attempting assessment tasks, and applying themselves in tutorial exercises, students will develop competency in mathematical presentation, written and verbal skills.
Relation to graduate attributes: The above outcomes are related to the development of the Science Faculty Graduate Attributes, in particular: 1.**Research, inquiry and analytical thinking abilities**, 4. **Communication**, 6. **Information literacy**

`https://moodle.telt.unsw.edu.au`

Detailed Course Descriptions

Overview: Optimization is an area of mathematics that directly deals with the problem of making the best possible choice from a set of feasible choices. It seeks to understand how we achieve the best possible choice and how we can use this knowledge to improve management and technical decisions in science, engineering and commerce. Thinking in terms of choices is common in our cognitive culture and searching for the best possible choice is a basic human desire.Thus models of optimization arise everyday as management and technical decisions in many areas of human activity.

Problems of engineering design (such as the design of electronic circuits subject to a tolerancing and tuning provision), information technology and data sciences (such as the extraction of meaningful information from large databases and the classification of data), financial decision making and investment planning (such as the selection of optimal investment portfolios), and transportation management and so on arise in the form of a multi-variable optimization problem or an optimal control problem.

Optimization has its foundation in the development of calculus by Newton and Leibniz in the 17^{th} century. The solution of large multi-variable optimization problems using computers started with the work of Dantzig in the late 1940s and 1950s on the simplex method for linear programming. Now, multi-variable optimization problems with hundreds of variables can be solved routinely.

The topics will be covered in the following order.

Week	Topics
Week 1 (Lec 1-4)	**Optimization —-What is it?**: Modelling; standard form formulations, norms, existence, relaxation, gradients and Hessians; positive definite matrices.
Week 2 (Lec 5-8)	**Convexity of Sets and Functions**: Convex stes, extreme points, convex combinations, convex functions, epigraphs, extrema of convex functions
Week 3 (Lec 9-12)	**Optimization: unconstrained & Equality constraints**: First order optimality principles; Second-order optimality principles; necessary conditions;sufficient conditions; convexity and global optimality conditions, Equality constraints, regularity conditions, method of Lagrange multipliers; first-and second-order optimality conditions
Week 4 (Lec 13-16)	**Optimization: inequality constraints, global optimality and duality**: KKT conditions Necessary and sufficient global optimality conditions; Strong and weak duality, right-hand side perturbations
Week 5 (Lec 17-20)	**Numerical Methods**: Rates of convergence, iterative methods, descent methods, line search methods; steepest descent methods
Week 6 (BREAK)	
Week 7 (Lec 21-24)	**Newton and conjugate gradient methods**: Basic Newton's methods conjugate gradient methods
Week 8 (Lec 25-28)	**Penalty methods & introduction to optimal control**: Basic Newton's methods penalty functions; optimal control models.
Week 9 (Lec 29-32)	**Optimal control problems & PMP**: systems of differential equations; Pontryagin maximum principle (PMP)
Weeks 10 & 11 (Lec (33-36)	**Applications of PMP**: Autonomous control problems, with fixed targets, free time problems, Extension of PMP to problems with general targets; non-autonomous problems.

Additional Resources and Support

- **Text and Reference Books**

  There is NO textbook which covers all aspects of this course. General reference books are detailed in the last section.

- **Tutorial Exercises and Videos**

  Problem sheets for tutorials will be provided via UNSW Moodle. These problems are for you to do to enhance mastery of the course.

  SOME of the problems will be done in tutorials, but you will learn a lot more if you try to do them before the tutorial.Tutorial videos will demonstrate how hard tutorial problems are solved.

- **Lecture Notes and Mini-Lecture Videos**

  A set of skeleton notes and summary sheets containing only definitions, theorems and proofs will be provided for SOME components of the course on UNSW Moodle.

  Mini-lecture videos will cover some background materials for the course and give proofs of key optimization theorems.

- **On-line Quizzess**

  On-line quizzes will give you multiple opportunities to practice the key concepts and techniques, learned in the lectures and tutorials, and provide instant feedback on answers.

- **Sample test videos**

  Sample test videos will cover some past class test questions.

- **Calculators**

You may bring your own UNSW approved Scientific Calculator to the class tests and the final exam. Calculators will not be provided for you.

- **ELISE (Enabling Library and Information Skills for Everyone)**

  ELISE is designed to introduce new students to studying at UNSW. Completing the ELISE tutorial and quiz will enable you to:

  - analyse topics, plan responses and organise research for academic writing and other assessment tasks
  - effectively and efficiently find appropriate information sources and evaluate relevance to your needs
  - use and manage information effectively to accomplish a specific purpose
  - better manage your time
  - understand your rights and responsibilities as a student at UNSW
  - be aware of plagiarism, copyright, UNSW Student Code of Conduct and Acceptable Use of UNSW ICT Resources Policy
  - be aware of the standards of behaviour expected of everyone in the UNSW community
  - locate services and information about UNSW and UNSW Library

  Some of these areas will be familiar to you, others will be new. Gaining a solid understanding of all the related aspects of ELISE will help you make the most of your studies at UNSW.

  The ELISE training webpages:
  `https://subjectguides.library.unsw.edu.au/elise/aboutelise`

- **Academic Skills Support and the Learning Centre**

  The Learning Centre offers academic support programs to all students at UNSW Australia. They assist students to develop approaches to learning that will enable them to succeed in their academic study. For further information on these programs please go to:
  `http://www.lc.unsw.edu.au/services-programs`

School and UNSW Policies

The School of Mathematics and Statistics has adopted a number of policies relating to enrolment, attendance, assessment, plagiarism, cheating, special consideration etc. These are in addition to the Policies of The University of New South Wales. Individual courses may also adopt other policies in addition to or replacing some of the School ones. These will be clearly notified in the Course Initial Handout and on the Course Home Pages on the Maths Stats web site.

Students in courses run by the School of Mathematics and Statistics should be aware of the School and Course policies by reading the appropriate pages on the Maths Stats web site starting at:
`https://www.maths.unsw.edu.au/currentstudents/assessment-policies`

The School of Mathematics and Statistics will assume that all its students have read and understood the School policies on the above pages and any individual course policies on the Course Initial Handout and Course Home Page. Lack of knowledge about a policy will not be an excuse for failing to follow the procedure in it.

- **Application for Special Consideration or Missed Assessments**

  Please adhere to the Special Consideration Policy and Procedures provided on the web page below when applying for special consideration.
  `https://student.unsw.edu.au/special-consideration`

  Please note that the application is not considered by the Course Authority, it is considered by a centralised team of staff at the Nucleus Student Hub.

  The School will contact you (via student email account) after special consideration has been granted to reschedule your missed assessment, for a lab test or paper-based test only.

For applications for special consideration for assignment extensions, please note that the new submission date and/or outcome will be communicated through the special consideration web site only, no communication will be received from the School.

For Dates on Final Term Exams and Supplementary Exams please check the "Key Dates for Exams" ahead of time to avoid booking holidays or work obligations.

`https://student.unsw.edu.au/exam-dates`

- **Course Evaluation and Development**

  Student feedback is very important to continual course improvement. This is demonstrated within the School of Mathematics and Statistics by the implementation of the UNSW online student survey myExperience, which allows students to evaluate their learning experiences in an anonymous way. myExperience survey reports are produced for each survey. They are released to staff after all student assessment results are finalised and released to students. Course convenor will use the feedback to make ongoing improvements to the course.

- **Academic Integrity and Plagiarism**

  UNSW has an ongoing commitment to fostering a culture of learning informed by academic integrity. All UNSW staff and students have a responsibility to adhere to this principle of academic integrity. Plagiarism undermines academic integrity and is not tolerated at UNSW. Plagiarism at UNSW is defined as using the words or ideas of others and passing them off as your own.

  The UNSW Student Code provides a framework for the standard of conduct expected of UNSW students with respect to their academic integrity and behaviour. It outlines the primary obligations of students and directs staff and students to the Code and related procedures. In addition, it is important that students understand that it is not permissible to buy essay/writing services from third parties as the use of such services constitutes plagiarism because it involves using the words or ideas of others and passing them off as your own. Nor is it permissible to sell copies of lecture or tutorial notes as students do not own the rights to this intellectual property.

  If a student breaches the Student Code with respect to academic integrity, the University may take disciplinary action under the Student Misconduct Procedure. The UNSW Student Code and the Student Misconduct Procedure can be found at:

  `https://student.unsw.edu.au/plagiarism`

  An online Module "Working with Academic Integrity" (https://student.unsw.edu.au/aim) is a six-lesson interactive self-paced Moodle module exploring and explaining all of these terms and placing them into your learning context. It will be the best one-hour investment you've ever made.

  Plagiarism: Plagiarism is presenting another person's work or ideas as your own. Plagiarism is a serious breach of ethics at UNSW and is not taken lightly. So how do you avoid it? A one-minute video for an overview of how you can avoid plagiarism can be found

  `https://student.unsw.edu.au/plagiarism`

- **Important Notes**

  If you believe your application for Special Consideration has not been processed, you should email specialconsideration@unsw.edu.au immediately for advice.

  If you suffer from a chronic or ongoing illness that has, or is likely to, put you at a serious disadvantage, then you should contact the Equitable Learning Services (previously known as SEADU) who provide confidential support and advice. They assist students:

  - living with disabilities
  - with long- or short-term health concerns and/or mental health issues
  - who are primary carers
  - from low SES backgrounds
  - of diverse genders, sexes and sexualities
  - from refugee and refugee-like backgrounds

- from rural and remote backgrounds
- who are the first in their family to undertake a bachelor-level degree.

Their web site is:

`https://student.unsw.edu.au/els/services`

Equitable Learning Services (ELS) may determine that your condition requires special arrangements for assessment tasks. Once the School has been notified of these, we will make every effort to meet the arrangements specified by ELS.

Additionally, if you have suffered significant misadventure that affects your ability to complete the course, please contact your Lecturer-in-charge in the first instance.

Reference Books

The general references on optimization are listed below. The standard of the references is somewhat higher than is required in MATH3161/MATH5165.

Optimization References: General references on multi-variable optimization include [1, 2, 5, 11] and on optimal control include [9, 12]

Linear Algebra and Differential Equations: Solving multi-variable optimization problems requires techniques from linear algebra, whereas solving optimal control problems requires solution methods of differential equations. An elementary treatment of linear algebra can be found in Strang [14], while a reference for differential equations is Zill [15].

Mathematical Software: Solving practical problems typically requires a computer software package like MATLAB [10] (see Pratap [13] for an introduction).

References

[1] F.J. Aragón-Artacho, M.A. Goberna, M.A. López, M.M.L. Rodriguez, Nonlinear Optimization, Springer, 2019.

[2] A. Beck, Introduction to Nonlinear Optimization – Theory, Algorithms and Applications with MATLAB, MOS-SIAM Series on Optimization. SIAM, 2014.

[3] D. P. Bertsekas Nonlinear programming: Second edition, Athena Scientific, Belmont, MA, 1999.

[4] J. E. Dennis and R. B. Schnabel, *Numerical Methods for Unconstrained Optimization and Nonlinear Equations*, SIAM Publications, Classics in Applied Mathematics, 1996.

[5] R. Fletcher, *Practical Methods of Optimization, 2nd Edition*, John Wiley, 2000.

[6] P. E. Gill, W. Murray, and M. H. Wright, *Practical Optimization*, Academic Press, New York and London, 1981.

[7] G. H. Golub and C. F. Van Loan, *Matrix Computations*, John Hopkins University Press, Baltimore and London, third ed., 1996.

[8] J. B. Hiriart-Urruty and C. Lemarechal, *Convex Analysis and Minimization Algorithms*, Springer-Verlag, Berlin, 1993

[9] L. M. Hocking, *Optimal Control: An Introduction to the Theory with Applications*, Oxford University Press, Oxford, 1991.

[10] MathWorks, *MATLAB & Simulink Student Version R2012A*, Englewood Cliffs, 2012. (See UNSW Bookshop `http://www.bookshop.unsw.edu.au/computing/`. The student edition includes the Optimization toolbox)

[11] J. Nocedal and S. J. Wright, *Numerical optimization*, Springer, (2nd edition) 2006.

[12] E. R. Pinch, *Optimal control and the calculus of variations*, Oxford University Press, Oxford, 1995.

[13] R. Pratap, *Getting started with MATLAB: A Quick Introduction for Scientists and Engineers*, Oxford University Press, 2009.

[14] G. Strang, *Linear Algebra and its Applications*, Harcourt Brace Jovanovich, San Diego, 3 ed., 1988.

[15] D. G. Zill, *Differential equations with boundary-value problems*, Second Edition, PWS-Kent Publishing company, Boston, 1989.