UNSW SCIENCE

SCHOOL OF MATHS AND STATISTICS

MATH3611

HIGHER ANALYSIS

Term 2, 2019

Cricos Provider Code: 00098G

MATH3611 – Course Outline

Information about the course

Course Authority/Lecturer: Dr Pinhas Grossman
RC-6112a, email `p.grossman@unsw.edu.au`.

Consultation: Time TBA.
Credit: This course counts for 6 Units of Credit (6UOC).
Prerequisites, Exclusions, etc:

- The prerequisites for MATH3611 are 12 UOC of Level 2 Mathematics including MATH2111 or MATH2011(CR), and an average mark of at least 70, or permission from the lecturer. It is highly recommended that students have done at least some of their Level 2 courses at the Higher level, and/or have passed MATH2701. Those students who feel that they are borderline in meeting these prerequisites should discuss their position with the lecturer and submit their first assignment early for review.
- Exclusions: MATH3570 Foundations of Calculus.
- MATH3611 is a prerequisite for all students intending to do Honours in Pure Mathematics.

Lectures: This course will meet for five hours per week during weeks 1-6 and 8-10. These five hours will be scheduled in two two-hour blocks and one one-hour-block. During week 7 there will only be a tutorial. Monday of Week 2 is a public holiday, so there will be no lecture on that day. There will be a two-hour lecture on Monday of Week 11.
Note: The week 6 and 7 schedule differs from the official timetable. The location for the Friday Week 6 lecture (which is not in the timetable) will be announced later.

Lecture 1	2 hours	Monday 12:00-14:00	Mathews Theatre D
Lecture 2	1 hours	Wednesday 15:00-16:00	Red Centre Theatre
Lecture 3	2 hours	Friday 16:00-18:00	Red Centre Theatre

More about tutorials:
One of the weekly meeting hours will be used as a tutorial. Note that it is expected that students will participate in tutorials more actively than is sometimes the case in first and second year. It is vital that you have attempted the tutorial problems before the tutorial, but the questions you raise should not be restricted to those problems. If there is some concept that you don't quite understand, ASK!
Moodle: Course material such as problem sheets and course notes will be available on Moodle.

Course aims

Real analysis is a central pillar of modern mathematics, and we will cover its foundations. We start with the concepts of limits and continuity, which are at the core of calculus, and we extend these concepts to quite general situations. The simplest case ('metric spaces') is when there is some way of measuring the distance between two points in the space. In the most important examples, a metric space occurs as a set of functions, so we will look at ways in which one might say that a sequence of functions converges. Taking these ideas one step further, we look at convergence which does not come from a generalized distance function. These are the ideas of point set topology. The course will also include topics such as countability, continuity, uniform convergence, and compactness and connectedness. As time permits, we may discuss additional topics such as measure and integration theory, Banach and Hilbert spaces, and linear operators. This is not a 'computational' course, but rather one in which you will develop your ability to think and write abstractly, precisely and creatively.

Detailed course schedule

It is intended that the following topics will be covered. The actual number of lectures per topic may vary from that given here!

(1) Basics of set theory, (3 hours)
Cardinality, countability
The Schroeder-Bernstein Theorem

(2) Metric spaces (9 hours)
Examples including sequence spaces and $C[0,1]$.
Topological properties of sets, open and closed sets, Convergence, completeness, closure and density, Normed spaces and inner product spaces. Contraction mapping theorem, Picard's theorem from ODEs.

(3) Sequences and series of functions on metric spaces (3 hours)
Pointwise and uniform convergence of sequences of functions.
Weierstrass M-test
Differentiation and integration of limits and sums.

(4) Topological spaces (9 hours)
Open and closed sets, bases for topologies, separability and countability
Convergence, Hausdorff topologies, generalised sequences
Continuity and homeomorphisms
Examples including weak topology, topology of local uniform convergence,

(5) Compactness and connectedness (8 hours)
Review of Heine-Borel

Continuous images of compact sets, compactness for metric spaces
Uniform continuity, Weierstrass' approximation theorem
Arzela's theorem. Path-connectedness, continuous images of connected sets

(7) The basics of measure and integration (3 hours)
Basic measure theory, Lebesgue measure
Integration, dominated convergence theorem.

(8) Hilbert space (3 hours)
Orthonormal bases
Review of examples
Generalising weak convergence: dual spaces;

Assessment

Assessment in this course will consist of 2 short assignments (worth 10% each), a slightly longer assignment (worth 20%), and a 2 hour final examination (60%). Further details of grading criteria are given at the end of this document.
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Assignments

Due dates for these are:

Task	Date Due	Weighting
Ass 1	Week 2	10%
Ass 2	Week 5	10%
Main Ass 3	Week 9	20%
Exam		60%

Assignments will give an opportunity for students to try their hand at more difficult problems requiring more than one line of argument and also introduce them to aspects of the subject which are not explicitly covered in lectures. They will enable students to consolidate their understanding of topics as we go which will ensure that later material does not become inaccessible.
Resist the urge to turn too quickly to Dr Google! In general I will try to avoid setting questions for which there is an answer easily found on the internet, but the main point is that the assignment questions are there to help you get your head around using the abstract definitions that we use in advanced mathematics. Putting in a bit more brain effort in the assignments will pay off when it comes to doing the exam! If you are really stuck, your lecturer may be able to get you started in the right direction.
Assignments must be written up using a high standard of presentation. Attention must be paid to spelling and grammar as well to as explaining the argument. All

assignments must be 'typed', that is, prepared using (preferably) LaTeX, Word or some other suitable package.
Late assignments will attract a significant late penalty, and will not be accepted after solutions have been posted.

Examination

The final examination will assess student mastery of the material covered in the lectures. The exam will include questions requiring knowledge of the definitions, examples and theorems in the course, as well as problems to be solved using such knowledge.
Duration: Two hours. **Weighting:** 60% of your final mark.
Further details about the final examination will be available in class closer to the time.

Additional resources and support

Problem Sheets

A set of problem sheets will be given out. These problems are for you to do to enhance mastery of the course.
An important ingredient in understanding the type of abstract mathematics that you will see in this course is acquiring a good set of concrete examples. That is, you won't fully understand the material just by knowing the definitions and theorems; you'll also need to know what metric spaces, nets, topologies, and so forth can look like. The problem sets will allow us to give you a greater exposure to such examples than we could cover in the lectures.
SOME of the problems will be done in tutorials, but you will learn a lot more if you try to do them before the tutorial. Some of the problems will be chosen as assignment questions for the fortnightly assignments.

Lecture notes

Some notes may be provided on the course website from time to time.

Textbooks

The set textbook for this course is:

- A.N. Kolmogorov and S.V. Fomin: Introductory Real Analysis (Dover, 1970; Call number: P517.5/125).

This covers most of the material for this course and also for some of the later analysis courses (such as Functional Analysis and Measure, Integration and Probability). It has the benefit of being quite cheap, but the disadvantage of sometimes using slightly

old-fashioned terminology. In case of a conflict, the 'official' definition is the one given in lectures!
Buying the text is not absolutely necessary, but you should find it a useful source of examples, explanations and ideas.
Other recommended books:

- G.B. Folland: Analysis. Modern techniques and their applications.
- W. Rudin: Principles of Mathematical Analysis
- G. F. Simmons: Introduction to Topology and Modern Analysis

Course Evaluation and Development

The School of Mathematics and Statistics evaluates each course each time it is run. We carefully consider the student responses and their implications for course development. It is common practice to discuss informally with students how the course and their mastery of it are progressing.

Student Learning Outcomes

Students taking this course are expected to:
(1) Demonstrate mastery of the definitions, theorems, and concepts of basic mathematical analysis, including the theory of metric, topological, and Banach spaces;(2) Demonstrate an ability to apply abstract mathematical theory to concrete prob-lems; (3) Demonstrate effective communication of mathematical reasoning, particularly in the writing of proofs and arguments.
These outcomes will be measured by the assessments, with the assignments focusing especially on (3) and the final exam focusing especially on (1).
Relation to graduate attributes: The above outcomes are related to the development of the Science Faculty Graduate Attributes, in particular: 1.**Research, inquiry and analytical thinking abilities**, 4. **Communication**, 6. **Information literacy**

Teaching strategies underpinning the course

New ideas and skills are introduced and demonstrated in lectures, then students develop these skills by applying them to specific tasks in problem sheets and assess-

ments.
Rationale for learning and teaching strategies: We believe that effective learning is best supported by a climate of inquiry, in which students are actively engaged in the learning process. Hence this course is structured with a strong emphasis on problem-solving tasks in assessment tasks, and students are expected to devote the majority of their class and study time to the solving of such tasks.
Assessment criteria: The main criteria for marking all assessment tasks will be clear and logical presentation of correct solutions.
Knowledge and abilities assessed: All assessment tasks will assess the learning outcomes outlined above.

Administrative matters

Details of the general rules regarding attendence, release of marks, special consideration etc are available via the School of Mathematics and Statistics Web page. You should particularly check the Student Services page at
`http://www.maths.unsw.edu.au/currentstudents/student-services`

Equity and diversity issues

Students with any special needs and requests should either see Markie Lugton Student Services Room 3072 and ensure that SEADU have registered you for provisions.
Similarly, any student whose study is being adversely affected by external issues should see the lecturer and/or get an appropriate counsellor/doctor/. . . to contact the lecturer.

Plagiarism and academic honesty

Plagiarism is the presentation of the thoughts or work of another as one's own. Issues you must be aware of regarding plagiarism and the university's policies on academic honesty and plagiarism can be found at

`http://student.unsw.edu.au/plagiarism`

You should be also be aware that plagiarism rules vary significantly from discipline to discipline. Some issues that are mathematics specific are discussed below.

Plagiarism in mathematics
The plagiarism rules **do not** preclude you from consulting references and discussing questions with other people. Indeed you can often learn a great deal from such discussions. It just means that you need to be clear and honest about where the ideas came from. This can usually be covered by a sentence such as 'This proof is based on the proof of Theorem 3.2.1 in . . . ' or 'The clever idea to consider random subsets of Γ was provided by Fred'.

Note in particular that:

- **You will not lose marks because you found the answer in a book (great research!) or were helped by a friend.**
- **You will lose marks if you don't say where you found a solution, or who helped you, or if you copy something without understanding what you have written.**

You don't need to reference definitions, theorems etc that you would expect everyone in the course to know (eg the Cauchy Integral formula, or the definition of an analytic function, or something from the lectures). On the other hand, if your proof depends on a slightly more unusual theorem (eg Jensen's formula), then you should (a) check that that theorem doesn't depend on the thing that you have been asked to prove, and (b) give a reference to where you found the result. Your solutions should be written so that a fellow student could read and follow what you have done — keep this in mind as you are writing up your work.

Obviously straight copying of someone else's assignment is both unethical and easily detected. You should never provide your written solutions to another student; inevitably their work will look just like yours and you'll both be penalized. If you work on a problem with a friend, make sure that you each write it up quite separately (**and mention that you discussed the problem with your classmate**).

Similarly, you should never copy word-for-word something that find in a text/webpage/.... In mathematics there is a limit to how much one can rearrange a proof to make it look like yours, so we allow greater degrees of similarlity than would be acceptable in many other disciplines. However it is almost never the case that you find a proof that has the same sort of notation that we are using, exactly the same definitions and theorems etc. You always need to adapt these things to the context of the course that you are doing.

It is **very** easy to tell when someone has copied something and not understood it. (And remember, if you found the solution via a web search, we can find it too!)

Serious cases of plagiarism will attract severe penalties.

Grading criteria

Roughly speaking, here is what I expect for the various grades:

Grade	Standard
PS	Knowledge of the main concepts, definitions and theorems. Ability to produce a simple proof using these, given time.
CR	Shows undertanding of the main concepts, definitions and theorems. Knows some examples. Can make a decent attempt at more complex proofs in assignments and can manage simple proofs under exam conditions.
DN	No major misunderstandings of the concepts and a good knowledge of main examples. Only misses more subtle points in most proofs in assignments and can have a decent go at harder proofs in the exam.
HD	Excellent understanding of the concepts and examples in the course. Proofs mainly watertight, especially in the assignments. For a top HD a level of mathematical flair and sophistication is expected.

Depending on the ability of the class, the exam (and to a lesser extent the main assignment) will be set to best allow me to test the above things and to differentiate between students. The performance bands in the exam will be mapped to the appropriate standard mark ranges (eg PS 50—64, HD 85—100) before being added to the assignment marks to produce a preliminary mark.

We may do a small degree of final scaling to the preliminary mark to avoid grade boundaries ending up in unfortunate places and to ensure consistency between courses.