Faculty of Science
School of Mathematics and Statistics

MATH5615

BANACH ALGEBRAS

Semester 2 2013

CRICOS Provider No: 00098G

MATH5615 – Course Outline

Information about the course

Course Authority: Pinhas Grossman

Lecturer: Pinhas Grossman RC-6112A, email `p.grossman@unsw.edu.au`.

Consultation: Please use email to arrange an appointment.

Credit & Prerequisites:

This course counts for 6 Units of Credit (6UOC).

MATH5605 is assumed knowledge for this course.

Lectures: The time and location of the lectures remain to be determined.

Course aims

Banach algebras have a lot of structure, combining the topological features of a Banach space with the algebraic features of a ring. Although we shall see some other examples, our main focus will be on examining Banach algebras consisting of continuous linear operators on Hilbert and Banach spaces.

The most powerful theorems in this area concern Banach algebras with some additional structure. A C^*-algebra is a Banach algebra with an adjoint operation. We shall first look at a characterization of commutative C^*-algebras, before briefly discussing the general case.

The main aim of the course is to use the characterization of commutative C^*-algebras to prove the Spectral Theorem for Normal Operators. Already you will have seen results about the 'diagonalization' of normal matrices and at least compact self-adjoint operators. The Spectral Theorem will allow us to give a general infinite dimensional version of these diagonalization results.

If time permits we shall look at some additional topics, possibly including basic properties of C^*-algebras and von Neumann algebras.

Course schedule

The actual path we shall take through the material will depend a little on the background and progress of the participants. The following is a preliminary guess:

1. General Introduction: Algebra, topology and so forth. Examples of Banach spaces and linear operators
2. Banach algebras: definitions and examples
3. Reminders of some basic functional analysis: Weak topologies and spectrum
4. Functional calculus
5. Abelian Banach algebras: the maximal ideal space and the Gelfand transform. Applications to harmonic analysis, the group algebra.
6. C^*-algebras: definitions and examples, representations of abelian C^*-algebras.
7. The spectral theorem for normal operators
8. Further topics in operator algebras

Assessment

Overview:

Task	Due Date	Weighting
Assignment 1	week 3	10%
Assignment 2	week 6	20%
Assignment 3	week 10	20%
Exam		50%
Total		100%

Assignments: Assignments will give an opportunity for students to try their hand at more difficult problems requiring more than one line of argument and also introduce them to aspects of the subject which are not explicitly covered in lectures. We shall have one short assignment due early in the course to get you started and then two more substantial assignments due in weeks 6 and 10. (Assignments will be due at the start of the last lecture for that week.) Draconian late penalties will apply at the whim of the lecturer.

Exam: The final two hour examination will assess student mastery of the material covered in the lectures. The exam will be worth 50% of your final mark. Further details about the final examination will be available in class closer to the time.

Textbooks

We shall initially mostly follow the first two chapters in

1. W.B. Arveson, *A Short Course in Spectral Theory.*

There are a very large number of other books in the library that include material that may be useful (try around 517.5 in the library). Some that you might look at are:

2. J.B. Conway, *A Course in Functional Analysis.*
3. *N. Dunford and J.T. Schwartz, Linear Operators, Part I, General Theory, Wiley Interscience, 1958. (The classic reference in this area.)*
4. *P.R. Halmos, A Hilbert Space Problem Book, (2nd ed.), Springer-Verlag, 1982.*
5. *R. Kadison and J.R. Ringrose, Fundamentals of the Theory of Operator Algebras, Vol. 1, Academic Press, 1983.*
6. *G.K. Pedersen, Analysis Now, Springer-Verlag, 1989.*

As the course progresses further references are likely to be given.

Course Evaluation and Development

The School of Mathematics and Statistics evaluates each course each time it is run. We carefully consider the student responses and their implications for course development. It is common practice to discuss informally with students how the course and their mastery of it are progressing.

Student Learning Outcomes

Students taking this course will develop an appreciation of the basic concepts of Functional Analysis, including the study of operator theory and the study of topological function spaces. These methods will be useful for further study in a range of other fields, e.g. Quantum Theory, Stochastic calculus and Harmonic analysis.

Relation to graduate attributes: The above outcomes are related to the development of the Science Faculty Graduate Attributes, in particular: 1.**Research, inquiry and analytical thinking abilities**, 4. **Communication**, 6. **Information literacy**

Teaching strategies underpinning the course

New ideas and skills are introduced and demonstrated in lectures and problem sessions, then students develop these skills by applying them to specific tasks in problem sheets and assessments.

Rationale for learning and teaching strategies

We believe that effective learning is best supported by a climate of inquiry, in which students are actively engaged in the learning process. Hence this course is structured with a strong emphasis on problem-solving tasks in assessment tasks, and students are expected to devote the majority of their class and study time to the solving of such tasks.

Rationale for Assignments: Assignments will give an opportunity for students to try their hand at more difficult problems requiring more than one line of argument and also introduce them to aspects of the subject which are not explicitly covered in lectures.

Rationale for Examinations: The final examination will assess student mastery of the material covered in the lectures.

Assessment criteria: The main criteria for marking all assessment tasks will be clear and logical presentation of correct solutions.

Knowledge and abilities assessed: All assessment tasks will assess the learning outcomes outlined above.

Administrative matters

Additional Assessment

This is at
`http://www.maths.unsw.edu.au/students/current/policies/addasspolicy.html`

School Rules and Regulations

Fuller details of the general rules regarding attendence, release of marks, special consideration etc are available via the School of Mathematics and Statistics Web page at `http://www.maths.unsw.edu.au/students/current/policies/studentpolicy.html`.

Plagiarism and academic honesty

Assignments must be YOUR OWN WORK, or severe penalties will be incurred. You should consult the University web page on plagiarism. Plagiarism is the presentation of the thoughts or work of another as one's own. Issues you must be aware of regarding plagiarism and the university's policies on academic honesty and plagiarism can be found at `http://www.lc.unsw.edu.au/plagiarism` and `http://www.lc.unsw.edu.au/plagiarism/plagiarism_STUDENTBOOK.pdf`.