Mathematics
and Statistics
UNSW Science

MATH5615

BANACH ALGEBRAS

Term 2, 2019

MATH5615 – Course Outline

Information about the course

Course Authority: Associate Professor Ian Doust

Lecturer: Ian Doust RC-6113, email `i.doust@unsw.edu.au`.

Consultation: Please use email to arrange an appointment.

Credit: This course counts for 6 Units of Credit (6UOC).

Prerequisites: MATH5605 is assumed knowledge for this course. (This means that you should be comfortable with Hilbert and Banach spaces, bounded operators on those spaces, and the standard topologies that arise in analysis, such as the norm topology, the weak topology and the topology of pointwise convergence. Having met the spectrum of a bounded operator and compact operators, and some of the main structures from algebra, would also be helpful.)

Lectures: There are 4 classes per week. These will mainly be lectures, but we will occasionally pause for a tutorial/review class.

Tuesday	3pm – 5pm	RC-3085
Thursday	10am – noon	RC-3085

Course aims

Banach algebras have a lot of structure, combining the topological features of a Banach space with the algebraic features of a ring. Although we shall see many other examples, our main focus will be on examining Banach algebras consisting of continuous linear operators on Hilbert and Banach spaces.

The most powerful theorems in this area concern Banach algebras with some additional structure. A C^*-algebra is a Banach algebra with an adjoint operation. We shall first look at a characterization of commutative C^*-algebras, before briefly discussing the general case.

Along the way we shall see that much of the theory of complex analysis actually extends to the case of Banach algebra valued functions, and this will provide us with a powerful tool throughout the course.

Our main goal is to get to the Spectral Theorem for Normal Operators, one of the central results of operator theory. This theorem, which is a generalization of the diagonalization theorem for normal matrices, tells us all about the structure and properties of normal operators on a Hilbert space

Syllabus

The actual path we shall take through the material will depend a little on the background and progress of the participants. The following is a preliminary guess:

1. General Introduction: Algebra, topology and so forth. Examples of Banach spaces and linear operators
2. Banach algebras: definitions and examples
3. Reminders of some basic functional analysis: Weak topologies and spectrum
4. Functional calculus
5. Abelian Banach algebras: the maximal ideal space and the Gelfand transform. Applications to harmonic analysis, the group algebra.
6. C^*-algebras: definitions and examples, representations of abelian C^*-algebras.
7. The spectral theorem for normal operators
8. Further topics in operator algebras

Assessment

Overview:

Task	Due Date	Weighting
Assignment 1	week 2	10%
Assignment 2	week 6	20%
Assignment 3	week 9	20%
Exam		50%
Total		100%

Assignments: Assignments will give an opportunity for students to try their hand at more difficult problems requiring more than one line of argument and also introduce them to aspects of the subject which are not explicitly covered in lectures. We shall have one short assignment due early in the course to give you some feedback on your how you are presenting your work, and to check that you are not mathematically out of your depth.

There are two more substantial assignments due in weeks 6 and 9 which will require you to attack some more challenging problems.

Assignments must be prepared in LaTeX or something similar[1]. They will be due at the start of the last lecture for that week.

Draconian late penalties will apply at the whim of the lecturer.

[1] Word if you must!

Exam: The final two hour examination will assess student mastery of the material covered in the lectures. The exam will be worth 50% of your final mark. Further details about the final examination will be available in class closer to the time.

Grading criteria

Roughly speaking, here is what I expect for the various grades:

Grade	Standard
PS	Knowledge of the main concepts, definitions and theorems. Ability to produce a simple proof using these, given time.
CR	Shows undertanding of the main concepts, definitions and theorems. Knows some examples. Can make a decent attempt at more complex proofs in assignments and can manage simple proofs under exam conditions.
DN	No major misunderstandings of the concepts and a good knowledge of main examples. Only misses more subtle points in most proofs in assignments and can have a decent go at harder proofs in the exam.
HD	Excellent understanding of the concepts and examples in the course. Proofs mainly watertight, especially in the assignments. For a top HD a level of mathematical flair and sophistication is expected.

Depending on the ability of the class, the exam will be set to best allow me to test the above things and to differentiate between students. The performance bands in the exam will be mapped to the appropriate standard mark ranges (eg PS 50—64, HD 85—100) before being added to the assignment marks to produce a preliminary mark. Closer to the end of session I will give the class some idea of what sort of expectations I have for raw marks in the final exam.

Notes and references

Lecture notes, problem sets and so forth for the course will be made available on Moodle. These are unlikely to match exactly what is said in lectures. In particular, the lectures are likely to contain different examples, and these examples are examinable!

If you like consulting textbooks, the ones we shall be following most closely are

1. W.B. Arveson, *A Short Course in Spectral Theory.* (Chapters 1 and 2)

2. J.B. Conway, *A Course in Functional Analysis. (Chapters 7, 8 and 9)*

There are a very large number of other books in the library that include material that may be useful (try around 517.5 in the library). Some that you might look at are:

3. N. Dunford and J.T. Schwartz, *Linear Operators, Part I, General Theory,* Wiley Interscience, 1958. (The classic reference in this area.)

4. P.R. Halmos, *A Hilbert Space Problem Book,* (2nd ed.), Springer-Verlag, 1982.

5. R. Kadison and J.R. Ringrose, *Fundamentals of the Theory of Operator Algebras, Vol. 1,* Academic Press, 1983.

Course Evaluation and Development

The School of Mathematics and Statistics evaluates each course each time it is run. We carefully consider the student responses and their implications for course development. It is common practice to discuss informally with students how the course and their mastery of it are progressing.

Student Learning Outcomes

Students taking this course will develop an appreciation of the basic concepts of Functional Analysis, including the study of operator theory and the study of topological function spaces. These methods will be useful for further study in a range of other fields, e.g. Quantum Theory, Stochastic calculus and Harmonic analysis.

Relation to graduate attributes: The above outcomes are related to the development of the Science Faculty Graduate Attributes, in particular: 1.**Research, inquiry and analytical thinking abilities**, 4. **Communication**, 6. **Information literacy**

Teaching strategies underpinning the course

New ideas and skills are introduced and demonstrated in lectures and problem sessions, then students develop these skills by applying them to specific tasks in problem sheets and assessments.

Rationale for learning and teaching strategies

We believe that effective learning is best supported by a climate of inquiry, in which students are actively engaged in the learning process. Hence this course is structured with a strong emphasis on problem-solving tasks in assessment tasks, and students are expected to devote the majority of their class and study time to the solving of such tasks.

Rationale for Assignments: Assignments will give an opportunity for students to try their hand at more difficult problems requiring more than one line of argument and also introduce them to aspects of the subject which are not explicitly covered in lectures.

Rationale for Examinations: The final examination will assess student mastery of the material covered in the lectures.

Assessment criteria: The main criteria for marking all assessment tasks will be clear and logical presentation of correct solutions.

Knowledge and abilities assessed: All assessment tasks will assess the learning outcomes outlined above.

Administrative matters

Special consideration and additional assessment

Be aware that the university's policies in this area have changed considerably. Note in particular that

- There is no longer any "Concessional Additional Assessment" granted to student who nearly passed.
- All Special Consideration requests are now handled by an on-line process which is processed outside the School of Mathematics and Statistics.
- UNSW now has a "Fit to Sit" policy on exams.

Please adhere to the University special consideration policy and procedures provided on the web page.

`https://student.unsw.edu.au/special-consideration`

For final exams with special consideration granted, the Exams Unit will email the rescheduled supplementary exam date, time and location to your student zID email account directly, so please regularly check your student email account (zID account).

The supplementary exam period/dates for the final exam can be found at this web site: `https://student.unsw.edu.au/exam-dates`. Please ensure you are aware of these dates and that you are available during this time.

For minor issues during the session it may be more appropriate to speak to the lecturer first to seek advice on whether to put in a formal special consideration request.

School Rules and Regulations

Fuller details of the general rules regarding attendence, release of marks, special consideration etc are available via the School of Mathematics and Statistics Web page at `http://www.maths.unsw.edu.au/students/current/policies/studentpolicy.html`.

Plagiarism and academic honesty

Plagiarism is the presentation of the thoughts or work of another as one's own.

You should consult the University web page on plagiarism. Plagiarism is the presentation of the thoughts or work of another as one's own. Issues you must be aware of regarding plagiarism and the university's policies on academic honesty and plagiarism can be found at `https://student.unsw.edu.au/plagiarism`.

You should be also be aware that plagiarism rules vary significantly from discipline to discipline. Some issues that are mathematics specific are discussed below.

Plagiarism in mathematics

The plagiarism rules **do not** preclude you from consulting references and discussing questions with other people. Indeed you can often learn a great deal from such discussions. It just means that you need to be clear and honest about where the ideas came from. This can usually be covered by a sentence such as 'This proof is based on the proof of Theorem 3.2.1 in ... ' or 'The clever idea to consider random subsets of Γ was provided by Fred'.

Note in particular that:

- **You will not lose marks because you found the answer in a book (great research!) or were helped by a friend.**
- **You will lose marks if you don't say where you found a solution, or who helped you, or if you copy something without understanding what you have written.**

You don't need to reference definitions, theorems etc that you would expect everyone in the course to know (eg the Intermediate Value Theorem, or the definition of an analytic function, or something from the lectures). On the other hand, if your proof depends on a slightly more unusual theorem (eg Jensen's formula), then you should (a) check that that theorem doesn't depend on the thing that you have been asked to prove, and (b) give a reference to where you found the result. Your solutions should be written so that a fellow student could read and follow what you have done — keep this in mind as you are writing up your work.

Obviously straight copying of someone else's assignment is both unethical and easily detected. You should never provide your written solutions to another student; inevitably their work will look just like yours and you'll both be penalized. If you work on a problem with a friend, make sure that you each write it up quite separately (**and mention that you discussed the problem with your classmate**).

Similarly, you should never copy word-for-word something that find in a text/webpage/.... In mathematics there is a limit to how much one can rearrange a proof to make it look like yours, so we allow greater degrees of similarlity than would be acceptable in many other disciplines. However it is almost never the case that you find a proof that has the same sort of notation that we are using, exactly the same definitions and theorems etc. You always need to adapt these things to the context of the course that you are doing.

It is **very** easy to tell when someone has copied something and not understood it. Remember, if you found the solution via a web search[2], we can find it too! And definitely don't try asking your assignment questions on Stackexchange etc! If you need some help, come and see me.

Serious cases of plagiarism will attract severe penalties and will be officially recorded by the University.

[2]While the internet is a very useful source of information, you should resist the urge to seek its advice too quickly. You learn a lot from trying hard to solve a problem from just the information given, and this is good training for attacking problems where the answer is not written down somewhere. Even if you end up seeking advice (via google, a book or a human), you will get much more out of this advice if you can appreciate how they got around the point where you got stuck.