Faculty of Science
School of Mathematics & Statistics

MATH5645
ALGEBRAIC NUMBER THEORY

Semester 2, 2017

MATH5645 – Course Outline

Information about the course

Course Authority/Lecturer:

Dr. Alina Ostafe, RC-4078
Email: alina.ostafe@unsw.edu.au
Consultation: Times will be announced in week 2.

Credit:

This course counts for 6 Units of Credit (6UOC).

Prerequisites:

The prerequisites for MATH5645 are MATH2501 or MATH2601 and at least two out of MATH3521, MATH3711 or MATH5706, with an average mark of at least 70.

Location and Times:

Monday	4 pm – 6 pm	RC3085
Thursday	2 pm – 3 pm	RC3085

There will be three hours of classes each week. There will be a tutorial every other week. All other classes will be lectures.

Course aims

Being at the intersection of two of the most beautiful and important areas of mathematics, *Algebra* and *Number Theory*, this course will build on your abstract algebra background to address number theoretic problems regarding integers or rational numbers, and their generalisation to algebraic numbers. As such, most of the course will develop the theory of *algebraic number fields*, that are finite extension of the set of rational numbers, with their many applications (such as solving diophantine equations) and links to other areas (such as Diophantine geometry, arithmetic dynamical systems, or even more applied one, such as cryptography).
The course will start with the theory of algebraic numbers and algebraic integers. Then, we will study ideals in rings of integers and their factorisation (different to what we are used in the case of the integers), followed by ramification theory which roughly describes the decomposition of prime ideals in a given number field when lifted to finite extensions. We will continue with ideal class groups and units, and if time allows, we will discuss also valuations and several applications to solving diophantine equations.

Besides regular material, at the end of each week we will briefly discuss some research topics. They will help to see a broader picture, gain more understanding and ignite more interest to this area. These topics may later be developed into independent research projects.

Detailed course schedule

It is intended that the following topics will be covered. The actual number of lectures per topic will be fixed along the course.

1. **Algebraic numbers and algebraic integers**
   Formal definitions of algebraic number fields and rings of integers
   Norms and traces
   Embeddings in $\mathbb{C}$
   Discriminant

2. **Ideals and prime decomposition in rings of integers**
   Factorisation of ideals
   Chinese remainder theorem
   Splitting of primes

3. **Galois theory and prime decomposition**
   Galois extensions
   The decomposition group and field
   The Frobenius automorphism

4. **Cyclotomic fields**
   Galois groups of cyclotomic fields
   Ramification theory for cyclotomic fields

5. **Ideal Class group and units**
   Ideal class group and unit group
   Dirichlet units theorem

6. **Valuations**
   Archimidean places
   Non-archimidean places
   Weak approximation

7. **Applications: Diophantine equations**
   Roth's theorem
   S-unit equations
   Integral points on curves (Siegel theorem)

Assessment

Assessment in this course will consist of

- **Weekly problem submissions, 10% in total**: Each week, a set of exercises will be made available, with one question designated for independent work and follow up submission. Each submission brings up to 1%, and can add up to 10% (this also means that if by some misfortune you missed 2 submissions out of 12, you can still gain the full 10% in total)

- **Two main assignments with oral presentations, 15% each**: Each assignment will be a small research project completed in a team of two to mimic real collaborative work in mathematics. Please note that although the tasks can (and should) be split between team members, each team member will share the responsibility for the entire project. You will also have to give a short ($\sim 7-8$ minutes) presentation in the class. You final grade will depend on both: your written submission and your presentation: evaluated for both the actual results reported and its clarity.

  Assignments must be written up using a high standard of presentation, using LaTeX. Using Word or some other package must be approved in advance in writing by the course leader.

- **Final examination, 60% in total**: The final examination will assess student mastery of the material covered in the lectures. The exam will last for 2 hours and include questions requiring knowledge of the definitions, examples and theorems in the course, as well as problems to be solved using such knowledge. **Duration:** Two hours. **Weighting:** 60% of your final mark.

  Further details about the final examination will be available in class closer to the time.

Due dates for these are:

Task	Date Due	Weighting
12 weekly problems	TBA	10%
Assignment and Presentation 1	TBA	15%
Assignment and Presentation 2	TBA	15%
Final Exam (2 hours)	TBA	60%

Additional resources and support

Problem Sheets

A set of problem sheets will be given out. These problems are for you to enhance mastery of the course.
An important ingredient in understanding the type of abstract mathematics that you will see in this course is acquiring a good set of concrete examples. You will not fully

understand the material just by knowing the definitions and theorems. The problem sets will allow us to give you a greater exposure to such examples than we could cover in the lectures.
Some of the problems will be discussed in the class, but you will learn a lot more if you try to solve them independently.

Lecture notes

Some notes may be provided on the course website from time to time.

Textbooks

The lectures will cover all the material that you need to know, but nevertheless, you will probably find it useful to supplement your studies by looking at texts such as those below.

- A. Fröhlich and M.J. Taylor, *Algebraic Number Theory*, Cambridge, 1993.
- S. Lang, *Algebraic Number Theory*, Graduate Texts in Mathematics, **110** (2 ed.), Springer-Verlag, New York.
- D. Marcus, *Number Fields*, Springer-Verlag, New York, 1977.
- I. N. Stewart and D.O. Tall, *Algebraic Number Theory and Fermat?s Last Theorem*, A. K. Peters, third edition, 2002.
- U. Zannier, *Diophantine Analysis*, Publ. Scuola Normale Superiore, Pisa, 2009.

Student Learning Outcomes

Students taking this course will develop an appreciation of the basic concepts of the theory of finite fields. They wil also get a glimpse of modern developments in the area and will get familiar with a diverse scope of applications. These methods will be useful for further study in a range of other fields, e.g. , Number Theory, Cryptography, Coding Theory and Theoretic Computer Science.
Relation to graduate attributes: The above outcomes are related to the development of the Science Faculty Graduate Attributes, in particular: 1.**Research, inquiry and analytical thinking abilities**, 4. **Communication**, 6. **Information literacy**

Teaching strategies underpinning the course

New ideas and skills are introduced and demonstrated in lectures, then students develop these skills by applying them to specific tasks in problem sheets and assessments.
Rationale for learning and teaching strategies: We believe that effective learning is best supported by a climate of inquiry, in which students are actively engaged in the

learning process. Hence this course is structured with a strong emphasis on problem-solving tasks in assessment tasks, and students are expected to devote the majority of their class and study time to the solving of such tasks.
Assessment criteria: The main criteria for marking all assessment tasks will be clear and logical presentation of correct solutions.
Knowledge and abilities assessed: All assessment tasks will assess the learning outcomes outlined above.

Administrative matters

Details of the general rules regarding attendance, release of marks, special consideration etc are available via the School of Mathematics and Statistics Web page. You should particularly check the Student Services page at
`http://www.maths.unsw.edu.au/currentstudents/student-services`
Students should particularly note that special additional assessment policies that apply to students doing MATH2xxx and MATH3xxx courses whose final mark is in the range 40–49.

Equity and diversity issues

Students with any special needs and requests should either see the lecturer as soon as possible, or ensure that SEADU sends appropriate documentation.
Similarly, any student whose study is being adversely affected by external issues should see the lecturer and/or get an appropriate counsellor/doctor/…to contact the lecturer.

Plagiarism and academic honesty

Plagiarism is the presentation of the thoughts or work of another as one's own.
Issues you must be aware of regarding plagiarism and the university's policies on academic honesty and plagiarism can be found at

`http://student.unsw.edu.au/plagiarism` , and

`http://my.unsw.edu.au/student/atoz/Plagiarism.html`.

You should also be aware that plagiarism rules vary significantly from discipline to discipline. Some issues that are mathematics specific are discussed below.

Plagiarism in mathematics

The plagiarism rules **do not** preclude you from consulting references and discussing questions with other people. Indeed you can often learn a great deal from such discussions. It just means that you need to be clear and honest about where the ideas came from. This can usually be covered by a sentence such as 'This proof is based on the proof of Theorem 3.2.1 in … ' or 'The clever idea to consider random subsets of Γ was provided by John Smith'.
Note in particular that:

- **You will not lose marks because you found the answer in a book or were helped by a friend (this is, how we all do in our research: search in the literature and talk to our colleagues).**
- **You will lose marks if you don't say where you found a solution, or who helped you, or if you copy something without understanding what you have written.**

You don't need to reference definitions, theorems etc that you would expect everyone in the course to know (eg Chinese Remainder Theorem, or the definition of an algebraic number, or something from the lectures). On the other hand, if your proof depends on a slightly more unusual theorem (eg, Kronecker-Weber Theorem), then you should (a) check that that theorem doesn't depend on the thing that you have been asked to prove, and (b) give a reference to where you found the result. Your solutions should be written so that a fellow student could read and follow what you have done — keep this in mind as you are writing up your work.

Straight copying of someone else's assignment is both unethical and easily detected. You must never provide your written solutions to another student; inevitably their work will look similar to yours and you both will be equally penalised. If you work on a problem with a friend, make sure that you each write it up quite separately (and mention that you discussed the problem with your classmate).

Similarly, you should never copy word-for-word something that find in a text/web-page/dots. In mathematics there is a limit to how much one can rearrange a proof to make it look like yours, so we allow greater degrees of similarity than would be acceptable in many other disciplines. However it is almost never the case that you find a proof that has the same sort of notation that we are using, exactly the same definitions and theorems etc. You always need to adapt these things to the context of the course that you are doing.

It is **very** easy to tell when someone has copied something and not understood it.

Serious cases of plagiarism will attract severe penalties.

Grading criteria

Roughly speaking, here is what I expect for the various grades:

Grade	Standard
PS	Knowledge of the main concepts, definitions and theorems. Ability to produce a simple proof using these, given time.
CR	Shows undertanding of the main concepts, definitions and theorems. Knows some examples. Can make a decent attempt at more complex proofs in assignments and can manage simple proofs under exam conditions.
DN	No major misunderstandings of the concepts and a good knowledge of main examples. Only misses more subtle points in most proofs in assignments and can have a decent go at harder proofs in the exam.
HD	Excellent understanding of the concepts and examples in the course. Proofs mainly watertight, especially in the assignments. For a top HD a level of mathematical flair and sophistication is expected.

The exam and the main assignments will be set to best allow me to test the above things and to differentiate between students. These will be adjusted to the ability of the class. The performance bands in the exam will be mapped to the appropriate standard mark ranges (eg PS 50—64, HD 85—100) before being added to the assignment marks to produce a preliminary mark. Closer to the end of session I will give the class some idea of what sort of expectations I have for raw marks in the final exam.
We may do a small degree of final scaling to the preliminary mark to avoid grade boundaries ending up in unfortunate places and to ensure consistency between courses.