UNSW AUSTRALIA

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COURSE OUTLINE

MATH5725

GALOIS THEORY

Semester 2, 2016

Course Outline

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Consultation Hours: TBA (see webpage)

Most of the information you need to know about the course can be gotten from the webpage above.

Lectures/Tutorials: The class times are Thursday 1-3 and Friday 3-4. There will be a tutorial every odd numbered week during the second hour of Thursday. All other classes will be lectures.

Lectures Notes: I will be posting lecture notes on my webpage. Boris Lerner has also typed up notes to a previous version of the course which you can also find there. This older course covers the material in greater depth, so may be of interest to you.

What you need to know to do this course

Ideal preparation for this course is a distinction or better in the MATH3711: Algebra course or equivalent. You need to be familiar with basic concepts in modern algebra and in particular, know a little about groups and field extensions. I hope the following notions are straightforward for you: normal subgroups, isomorphism theorems, group actions, minimal polynomials, algebraic extensions. The first problem sheet has some revision. If you can master the material there by week 2, you should be fine.

About this course

This course is a continuation of MATH3711 and, together with MATH5735: Modules and Representation Theory, rounds out the basic curriculum in undergraduate modern algebra. The material in this course is indispensable for anyone interested in number theory or algebra and is considered standard material for any pure mathematician.

The question which motivated Galois theory was: can you solve for the zeros of a polynomial in terms of the coefficients just using radicals, and the elementary operations of addition, subtraction, multiplication and division? The answer is yes for quartic and lower order polynomials, but no for higher order. The proof requires a detailed study of the symmetry of field extensions obtained by adjoining roots of polynomials. This symmetry is encoded in a group called the Galois group. The main aim of this course is to study field extensions via the Galois group. In the process, you will learn about fundamental concepts in algebra such as, solvable groups and the Galois correspondence. You will also see Galois's proof of the impossibility of solving the quintic via radicals.

Assessment

The assessment for this course has 5 components

Assessment task	Mark
Exam	65
Assignment 1	15
Assignment 2	15
Tutorial participation	4
Challenge questions	1

The assignments are meant to be relatively straightforward, once you have mastered the material. The hard part is of course understanding the material. It is expected that most of you will be getting close to full marks in the assignments. That way, a "pass" in the final exam should get you close to a credit. If you are having trouble with the assignments, you should talk to other students or to me about it. The most important thing is that you learn the material. The final exam will include questions of a more challenging nature and should distinguish the best students in the class.

I am experimenting with the flipped classroom this year and so I would like you all to present solutions to tutorial problems to each other. For each tutorial, you need to find a partner and pick a question not picked by another pair. You should inform me and the others of your question and then prepare to present your solution in class. (Hopefully, I'll be able to set up a forum on Moodle for this). If you are having difficulties, come and see me.

I will also be giving some challenge questions throughout the semester for those of you who are finding the material too easy. If you are the first to answer one of these, you can claim the Challenge question mark. If two of you answer the question corrrectly within a few days of each other, I'll also be happy to award the mark to both of you. I'll let you know when I'm no longer accepting answers to a question because I've received a correct answer.

Studying for this course

This course is fairly demanding conceptually, but hopefully, you will be getting used to this. The concepts will take a while to digest so don't expect to understand everything in lectures. Try to get as much as possible out of them, and go over the material regularly after class. I suspect that filling in these gaps in understanding will take up a significant amount of your study for this course. If you are understanding very little of the lectures, then that's probably an indication that you haven't properly understood material in earlier lectures. I also strongly suggest you supplement your learning by browsing the references below.

As you progress in your academic career, you will be given fewer and fewer exercises to assist your learning. Many of you will find this difficult, especially if your study in the past consisted of doing homework problems, and consulting the teacher when you are unable to do them. To help wean you off homework problems, you should check out my MATH3711 handout on my webpage entitled "Studying for this course".

You are at the stage in your academic career where your main aim should be to learn as much about mathematics as possible and not worry too much about marks. For me to give you the best mathematical education, I will occasionally cover material that is not examinable for various reasons. For example, it is important you know how the material in this course is related to those in other courses, some of which are not pre-requisites for this course though many of you will have done them.

Syllabus

The first 8 weeks of this course will cover basic material: the Galois correspondence, radical extensions, solvable groups and solvability by radicals. The last 4 weeks involve special topics which may include a selection from: infinite Galois theory, Galois descent, Kummer theory, the Galois correspondence in other parts of mathematics, Artin-Schreier theory, transcendence degree, transcendental numbers. The pace of exposition will pick up considerably in this second half.

Student learning outcomes

Mathematically, I hope you will consolidate your understanding of the fundamentals of modern algebra and gain an appreciation of how deep abstract mathematical concepts can be used to solve concrete problems.

From a skills perspective, I hope you will develop your problem solving and analytical skills. The course should also help you improve your conceptual thinking. You should understand by now, that modern mathematics is communicated in a very different fashion to other disciplines and to everyday speech. Though it can be terse, it has great precision. I hope in this course, you will gain a greater appreciation of the modes of mathematical communication.

References

The lectures will cover all the material that you need to know, but nevertheless, you will probably find it handy to supplement your studies by looking at texts such as those below. There are lots of texts designed for a first course in algebra. They vary a lot so you should scout around for what's suitable for you.

- Artin, Michael Algebra. Prentice Hall, Inc., Englewood Cliffs, NJ, 1991. xviii+618 pp. ISBN: 0-13-004763-5

- Bewersdorff, Jörg Galois theory for beginners. A historical perspective. Translated from the second German (2004) edition by David Kramer. Student Mathematical Library, 35. American Mathematical Society, Providence, RI, 2006. xx+180 pp. ISBN: 978-0-8218-3817-4; 0-8218-3817-2

- Cox, David A. Galois theory. Pure and Applied Mathematics (New York). Wiley-Interscience John Wiley & Sons, Hoboken, NJ, 2004. xx+559 pp. ISBN: 0-471-43419-1

- Escofier, Jean-Pierre Galois theory. Translated from the 1997 French original by Leila Schneps. Graduate Texts in Mathematics, 204. Springer-Verlag, New York, 2001. xiv+280 pp. ISBN: 0-387-98765-7

- Fraleigh, John B. A first course in abstract algebra. Addison-Wesley Publishing Co., Reading, Mass.-London-Don Mills, Ont. 1967

- Herstein, I. N. Abstract algebra. Third edition. With a preface by Barbara Cortzen and David J. Winter. Prentice Hall, Inc., Upper Saddle River, NJ, 1996. xviii+249 pp. ISBN: 0-13-374562-7

- Jacobson, Nathan Basic algebra. I. Second edition. W. H. Freeman and Company, New York, 1985. xviii+499 pp. ISBN: 0-7167-1480-9

- Kaplansky, Irving Fields and rings. Reprint of the second (1972) edition. Chicago Lectures in Mathematics. University of Chicago Press, Chicago, IL, 1995. x+206 pp. ISBN: 0-226-42451-0

- Stewart, Ian Galois Theory. Third edition. Chapman & Hall/CRC Mathematics. Chapman & Hall/CRC, Boca Raton, FL, 2004. xxxvi+288 pp. ISBN: 1-58488-393-6

- Stillwell, John Elements of algebra. Geometry, numbers, equations. Undergraduate Texts in Mathematics. Springer-Verlag, New York, 1994. xii+181 pp. ISBN: 0-387-94290-4

- Swallow, John Exploratory Galois theory. Cambridge University Press, Cambridge, 2004. xii+208 pp. ISBN: 0-521-83650-6; 0-521-54499-8

- Lang, Serge Algebra. Revised third edition. Graduate Texts in Mathematics, 211. Springer-Verlag, New York, 2002. xvi+914 pp. ISBN: 0-387-95385-X

- Weintraub, Steven H. Galois theory. Second edition. Universitext. Springer, NeW York, 2009. xiv+211 pp. ISBN: 978-0-387-87574-3

Continual Course Improvement

The School of Mathematics evaluates each course each time it is run. Feedback on the course is gathered using, among other means, UNSW's Course and Teaching Evaluation and Improvement Process. Following such feedback, I have now restructured the course so that applications of the main theory are presented as the theory unfolds rather than at the end.

School of Mathematics and Statistics Student Policies

School of Mathematics and Statistics policy regarding tests, assignments additional assessment etc can be found at

http://www.maths.unsw.edu.au/currentstudents/assessment-policies

The UNSW Plagiarism Policy is also there.

Daniel Chan