UNSW SCIENCE

SCHOOL OF MATHS AND STATISTICS

MATH5735

Modules and Representation Theory

Term 1, 2019

Information about the course

Course Authority/Lecturer: Jie Du, RC 4117, ph 93857087, j.du@unsw.edu.au

Credit, Prerequisites, Exclusions:

This course is part of Pure Mathematics honours and counts for 6 UOC.

A prerequisite is MATH3711 Higher Algebra or some similar first course in group theory; also basic linear algebra.

Lectures: Tuesdays 4-5 (4-6 for wk1), Fridays 2-4
Tutorials: Tuesdays 5-6 for weeks 2, 3, 4, 5, 7, 8, 9, 10, 11

Course Aims:

Performing linear algebra over a ring of scalars instead of a field leads to the notion of a module. The theory of modules is surprisingly subtle and has many applications, not only to other parts of mathematics but also to mathematical physics. This course focuses on the theory of modules over principal ideal domains and over semisimple
rings. The first has applications to linear algebra. The second has applications to group representations which is essential to the study of symmetry.

The course aims to develop key results in the classical theory of representations of finite groups and associated topics in module and ring theory. These results, mostly developed around 1900, have proved foundational for much of later algebra.

It follows on from earlier courses in linear algebra and group theory, and forms a basis for later graduate-level work in algebra.

Student Learning Outcomes

Students taking this course will develop an appreciation of the core of modern abstract algebra. The ability to deal with a high level of abstraction and to construct proofs of abstract results will be important. Nevertheless, the grounding of abstract concepts in examples will remain crucial – it is important to see how general concepts display themselves in small examples.

Regular attendance at lectures and work on the assignments will develop competency in listening, understanding and presentation skills.

Relation to graduate attributes: The above outcomes are related to the development of the Science Faculty graduate attributes, especially 1. Research, inquiry and analytical thinking, 2. Capability and motivation for intellectual development, and 3. Communication.

Teaching Strategies Underpinning the Course

New ideas and skills are introduced and demonstrated in lectures. Then students develop these skills by applying them to specific tasks in assignments. The course is taught in traditional lecture format, but rather more questions will be asked of students in class than is usual. That is designed to encourage a more active style of listening in students. Questions are welcome in or outside class.

Effective learning occurs when active listening in class is followed up by active effort at assignment questions. Effort should be devoted to understanding both the algebraic concepts and the methods of discovering and structuring proofs about them.

Assessment

Assessment will consist of an assignment, a quiz and a test (50%); final examination (50%).

Some initial collaborative work on assignments is permitted, for example discussion of initial ideas on how to solve problems or how to find relevant information. But the main work and write-up of assignments must be the student's own work.

Late assignments will be accepted with some small penalties for late submission.

Task	Date available	Date due	Form of submission	Weighting
Quiz	Week 4	Tutorial wk 4	Written	5%
Test	Weeks 6	Tutorial wk 6	Written	20%
Assignment	Friday, wk 6	Tutorial wk 9	Written	25%

Examination

Duration: Two hours

Rationale: The final examination will assess student mastery of all the material covered in lectures.

Weighting: 50% of final mark.

Additional Resources and Support

Lecture notes: There will not be printed notes provided, so it may be desirable to copy from the board. Some lecture notes from previous students are available:

Joel Beeren's notes of Daniel Chan's 2012 lectures.

Textbooks: There is no textbook, but Jacobson's *Basic Algebra I and II should be helpful.* Chapter 3 of I is a good introduction to the first parts of the course. Chapters 4 and 5 of II are good for semisimple algebras and group representations.

Informal help: Students are welcome to consult the lecturer in his office, RC 4113, whenever he is in.

Course Evaluation and Development

The School of Mathematics and Statistics evaluates each course each time it is run. We will discuss previous feedback at the beginning of the course.

Administrative Matters

Additional assessment: see http://www.maths.unsw.edu.au/students/current/policies/addasspolicy.html

School rules and regulations:

Fuller details of the general rules regarding attendance, release of marks, special consideration, etc. are available on the School's webpage at
http://www.maths.unsw.edu.au/students/current/policies/studentpolicy.html

Plagiarism and academic honesty:

Plagiarism is the presentation of the thoughts or work of another as one's own. Issues you must be aware of regarding plagiarism and the university's policies on academic honesty and plagiarism can be found at http://www.lc.unsw.edu.au/plagiarism .

Tentative Syllabus

1. Modules basics: submodules, homomorphisms, quotient modules, direct sums, free modules, torsion modules, isomorphism theorems
2. Simple modules, indecomposable modules, and Schur's lemma
3. Noetherian and artinian conditions, composition series
4. Jordan-Hoelder's Theorem
5. Finitely generated modules over a principal ideal domain, structure theorem
6. The structure of abelian groups and of linear transformations (Jordan canonical form)
7. Semisimple algebras and Wedderburn's Theorem
8. Group representations, character tables and orthogonality relations
9. Radicals of modules and rings, Fitting's lemma, Krull-Schmidt Theorem