# Faculty of Science
# School of Mathematics & Statistics

# MATH 5816

# CONTINUOUS TIME FINANCIAL MODELLING

## Semester 2, 2018

CRICOS Provider No: 00098G

**Information about the course**

**Course Authority and Lecturer:** Dr. Leunglung Chan,
RC-1036,
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**Consultation:** Please email me to arrange for an appointment.
**Units of Credit**: 6 UOC
**LECTURE TIME:** THURSDAYS 5-8pm Red Centre Central Wing 4082 (Weeks 1-12)

**Prerequisites**
MATH5965 Discrete Time Financial Modelling
MATH5975 Introduction to Stochastic Analysis

**Syllabus**

The main goal of the course is a detailed study of the classical Black-Scholes model and its variants. We introduce the concept of a continuously rebalanced portfolio, and we examine the arbitrage free property of the model by examining the existence and uniqueness of a martingale probability measure. We provide two alternative proofs of the Black-Scholes option pricing formula. The first relies on the calculation of the replicating strategy; it thus requires solving the Black-Scholes partial differential equation. The second method is based on probabilistic considerations and it makes direct use of the risk neutral valuation formula. We introduce and study the concepts of historical and implied volatilities. Subsequently, we present the approach known as the implied local volatility modelling. In this approach, the observed market prices at a given date (and thus the observed smiles and skews) are taken as inputs. In the second part of the course, we study contingent claims of American style in the Black-Scholes set up. We explain that the valuation of American claims is closely related to specific optimal stopping problems. We show that for the purpose of arbitrage valuation, the maximization of the expected discounted payoff should be done under the martingale measure. The last part of the course is devoted to cross-currency derivatives, stochastic volatility models and recent developments in continuous time financial modelling. Finally, we will look at stochastic optimal control based on the book of Pham (2009).

**Course structure**

- *The connection from the discrete-time model to the continuous-time model*: random walk, Brownian motion, information and sigma algebras, Markov property, reflection principle, quadratic variation, stochastic calculus such as Ito formula, stochastic integral.
- *Black-Scholes model and its extensions*: self-financing strategies, arbitrage opportunities, martingale measures, option pricing formula, replication of European contingent claims. Black-Scholes partial differential equation, completeness, Black's model for futures, Connections with PDEs, Risk-neutral pricing.
- *Exotic options*: Barrier options, Lookback options, Asian options.
- *American claims*: American call and put option, rational exercise time, early exercise premium representation, optimal exercise boundary.
- *Volatilities*: historical volatility, implied volatility, volatility surface, risk-neutral marginal distributions, local volatility models.
- *Cross-currency derivatives*: domestic and foreign martingale measures, currency forward contracts and options, change of Numeraire.
- Stochastic Volatility models
- Levy Models used in Finance
- Stochastic Optimal Control

**Course Aims**
The aim of MATH5816 is that at the end of session you should be able to understand the concepts and techniques discussed in the syllabus and be able to apply these concepts and techniques to appropriate problems in mathematical finance.

**Relation to other mathematics courses**
This is a compulsory course for the Master in Financial Mathematics. It builds on foundation knowledge of probability and stochastic analysis and will provide a solid background to the other core courses in the Master of Financial Mathematics, for example Math5985 and Math5925.

**Course Outcomes**
A student should be able

- to state definitions as specified in the syllabus

- to have working knowledge of appropriate theorems
- to apply the concepts and techniques of the syllabus to solve appropriate problems in mathematical finance
- to use specific and general results given specified assumptions
- to use terminology and reporting styles appropriately and successfully to communicate information and understanding

**Relation to graduate attributes**
The above outcomes are related to the development of the Science Faculty Graduate Attributes, in particular: 1. **Research, inquiry and analytical thinking abilities**, 2. **Communication**, 3. **Teamwork, collaborative and management skills**, 4. **Information literacy**

**Advice to Students**
Students are strongly advised to take note of the detailed syllabus and notes provided in lectures.
The level of depth of understanding required in this course is best understood by considering the examples given in lectures and assignments

**Teaching strategies underpinning the course**
New ideas and skills are introduced and demonstrated in lectures, then students develop these skills by applying them to specific tasks in assessments. In particular, Math5816 is taught through carefully planned lectures that logically develop the concept and techniques specified in the syllabus. Examples are emphasised as they provide the underlying motivation for the course, and because student best understand the general theory when it developed by more complex examples.
Students are encouraged to give constructive feedback during the teaching session. They are encouraged to work collaboratively with other students to develop their understanding and their problem solving skills.

**Rationale for learning and teaching strategies**
We believe that effective learning is best supported by a climate of enquiry, in which students are actively engaged in the learning process. To ensure effective learning, students should participate in class as outlined below.
We believe that effective learning is achieved when students attend all classes, have prepared effectively for classes by reading through previous lecture notes.
Furthermore, lectures should be viewed by the student as an opportunity to learn, rather than just copy down lecture notes.
Effective learning is achieved when students have a genuine interest in the subject and make a serious effort to master the basic material.

The art of logically setting out mathematics is best learned by watching an expert and paying particular attention to detail. This skill is best learned by regularly attending classes.

**Assessment**
Assessment in this course will consist of two assignments (10% each), a mid-session test (20%) and a final examination (60%).

**NOTE:** You will need to pass the final examination to pass the course.

**Knowledge and abilities assessed:** All assessment tasks will assess the learning outcomes outlined above, specifically, the ability to provide logical and coherent proofs of results and specific problems related to stochastic processes.

**Assessment criteria:** The main criteria for marking all assessment tasks will be clear and logical presentation of correct solutions. Please cite where you found your solution when necessary.

**Assignments**
**Rationale:** Assignments will give an opportunity for students to try their hand at more difficult problems requiring more than one line of argument and also introduce them to aspects of the subject which are not explicitly covered in lectures.

You can work in GROUPS OF UP TO 4 people. Hand in one copy of the solutions with ALL the names and student ID numbers on it.

Assignments must be handed in by the due date and time. Late submission will not be accepted unless there is documentary evidence of mitigating circumstances.

Each assignment must include a signed declaration of the plagiarism coversheet which can be found in Moodle. Every one in the group must sign the plagiarism coversheet.

All work submitted for assessment (other than formal examination scripts) will be returned with comments on the assessment where appropriate.
Assignments must be YOUR OWN GROUP WORK, or severe penalties will be incurred.

You should consult the University web page on plagiarism

`www.lc.unsw.edu.au/plagiarism`

| Task | Date Avail. | Date Due | Form of Submission | Weighting |
|---|---|---|---|---|
| Assignment 1 | Week 4 | Week 6 in class | Written | 10% |
| Assignment 2 | Week 9 | Week 11 in class | Written | 10% |

**Mid-session Test**
**Duration:** 90 minutes.
**Rationale:** The mid-session test will assess student mastery of the material covered in the first half of the course. It will be held in Week 7, Thursday at 5:00 pm. You will need to bring your own calculator.
**Weighting:** 20% of your final mark.
Further details about the mid-session test will be available in class closer to the time and also on Moodle.

**Final Examination**
**Duration:** Two hours.
**Rationale:** The final examination will assess student mastery of the material covered in the lectures.
**NOTE:** You will need to pass the final examination to pass the course.
**Weighting:** 60% of your final mark.
Further details about the final examination will be available in class closer to the time and also on Moodle. Please note the Supplementary Exams for S2 2018, for both final exam special consideration applications and those students who achieved a final mark from 45-49, will be scheduled between Saturday 8th December to Saturday 15 December 2018.

**Additional resources and support**

**Lecture notes**
Professor Marek Rutkowski's lecture notes will be provided on Moodle to supplement the class notes.

**Recommended Textbooks**
The class notes are based on Shreve's book for the geometric Brownian motion case, Jeanblanc for the Levy processes and Pham for stochastic optimal control.

- Steven Shreve: *Stochastic Calculus for Finance II. Continuous Time Models.* Springer, 2004.
- Marek Musiela and Marek Rutkowski: *Martingale Methods in Financial Modelling.* 2nd ed. Springer, 2005.
- Robert J. Elliott and Peter E. Kopp: *Mathematics of Financial Markets.* Springer, 1999.
- Alison Etheridge: *A Course in Financial Calculus*, Cambridge University Press, 2002.
- Rose-Anne Dana and Monique Jeanblanc: *Financial Markets in Continuous Time.* Springer, 2003.

- Huyèn Pham, *Continuous Time Stochastic Control and Optimization with Financial Applications*, 2009. This is an ebook in the our library.

**Course Evaluation and Development**
The School of Mathematics and Statistics evaluates each course each time it is run. We carefully consider the student responses and their implications for course development. It is common practice to discuss informally with students how the course and their mastery of it are progressing.
**Administrative matters**
**School Rules and Regulations**
Fuller details of the general rules regarding attendance, release of marks, special consideration etc are available via the School of Mathematics and Statistics Web page at
`http://www.maths.unsw.edu.au/students/current/policies/studentpolicy.html`.

**Plagiarism and academic honesty**

Plagiarism is the presentation of the thoughts or work of another as one's own. Issues you must be aware of regarding plagiarism and the university's policies on academic honesty and plagiarism can be found at
`http://www.lc.unsw.edu.au/plagiarism` and
`http://www.lc.unsw.edu.au/plagiarism/plagiarism_STUDENTBOOK.pdf`.