MATH5975

INTRODUCTION TO
STOCHASTIC ANALYSIS

Semester 1, 2015
MATH5975 – Course Outline

Information about the course

Course Authority and Lecturer: Dr Donna Mary Salopek
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Consultation: Consultation times are by appointment. Please speak to the lecturer in class, or send an email, to arrange for an appointment.

Credit: This course counts for 6 Units of Credit (6UOC).

Lectures: Mondays 5.00-8.00 pm in RC 3085.

There are no classes in the midsession break from the 3rd to 12th April, 2015.

Moodle: There is no formally written notes for this course. I will be writing the lecture notes on the blackboard in class only. If you miss a class, try to get the material from a fellow classmate. I will provide Professor Goldys notes from 2009 on Moodle in Week 2 or 3.

Assignments, problems sets and announcements will be provided via Moodle.

Course aims

Modern theory of financial markets relies on advanced mathematical and statistical methods that are used to model, forecast and manage risk in complex financial transactions. After the publication in 1973 of the ground-breaking paper of Black and Scholes on the arbitrage pricing of European call options, it became clear that Stochastic Analysis is an indispensable tool for the theory of financial markets, derivation of prices of standard and exotic options and other derivative securities, hedging related financial risk, as well as managing the interest rate risk.

In this course, you will learn the basic concepts and techniques of Stochastic Analysis, such as: Brownian motion, martingales, Itô stochastic integral, Itô’s formula, stochastic differential equations, equivalent change of a probability measure, integral representation of martingales with respect to a Brownian filtration, relations to second order partial differential equations, the Feynman-Kac formula, and jump processes.

Some concepts will be illustrated by examples relevant for financial applications. However, the main goal of the course is to provide a necessary mathematical background for MATH5816 Continuous Time Financial Modelling and MATH5985 Term Structure Modelling, rather then to focus directly on financial concepts.
I must stress that this course is a theorem-proof type of course. The aim to provide you with the necessary tools in stochastic analysis.

The course is a prerequisite for MATH5816 Continuous Time Financial Modelling and MATH5985 Term Structure Modelling and will provide a solid background for your Master Project as well.

Students taking this course will develop an appreciation of the basic problems of stochastic analysis and be able to

- Recognise which analysis procedure is appropriate for a given research problem
- Apply probability theory and stochastic analysis to practical problems
- Understand the usefulness of Stochastic Analysis in your professional area.

The ability to provide logical and coherent proofs of theoretic results, and the ability to solve problems via abstract methods will be paramount.

Through regularly attending lectures and applying themselves, students will develop competency in mathematical presentation, written and verbal skills.

New ideas and skills are introduced and demonstrated in lectures, then students develop these skills by applying them to specific tasks in assessments.

We believe that effective learning is best supported by a climate of enquiry, in which students are actively engaged in the learning process. To ensure effective learning, students should participate in class by attending all classes if possible; reading through previous lecture notes, and reading additional textbooks on stochastic analysis; viewing the lectures as an opportunity to learn, rather than just copy down lecture notes; and having a genuine interest in the subject and making a serious effort to master the basic material.

The above outcomes are related to the development of the Science Faculty Graduate Attributes, in particular: research, inquiry and analytical thinking abilities, communication, and information literacy.

## Assessment

UNSW assesses students under a standard based assessment policy. For how this policy is applied in the School of Mathematics and Statistics please see


Math 5975 assessment will consist of two assignments (10% each), midsession test (20%) and a final examination (60%).

For each assessment task, the main marking criteria will be clear and logical presentation of correct solutions.
Assignments

Rationale: Assignments will give an opportunity for students to try their hand at more difficult problems requiring more than one line of argument and also introduce them to aspects of the subject which are not explicitly covered in lectures.

Assignments must be YOUR OWN WORK, or severe penalties will be incurred. However, you can work in groups up to 4 students.

You should consult the University web page on plagiarism: www.lc.unsw.edu.au/plagiarism

Late assignments will not be accepted (unless the delay is justified by serious reasons).

Each assignment will be available at least 7 days before it is due to be submitted. In particular, the first assignment will be available on Moodle in Week 3 and due in class in Week 5. The second assignment will be available between weeks 9 and 10 and due in class in Week 12.

Weighting: Each assignment will be worth 10% of your final mark.

Midsession Test

Duration: 90 minutes on April 13th, 2015 at 5pm.

The midsession test will assess student mastery of the material covered in the lectures for the first half the course.

Weighting: 20% of your final mark

Further details about the midsession test will be available in class closer to the time.

Final Examination

Duration: Two hours and the time is set by UNSW. Please consult your myunsw for the time and place.

Rationale: The final examination will assess student mastery of the material covered in the lectures. You will need to pass the final examination to pass the course, otherwise you will get an UF - Unsatisfactory Fail (Unsatisfactory performance in an essential component of the course).

Weighting: 60% of your final mark.

Further details about the final examination will be available in class closer to the time.
Additional resources and support

Lecture Notes and Textbooks

There is no formally written notes for this course. I will be writing the lecture notes on the blackboard in class only. If you miss a class, try to get the material from a fellow classmate. I will be provide Professor Goldys notes from 2009 on Moodle in Week 2 or 3.

Suggested Readings:


Moodle

Some course materials such as assignments and announcements will be available on Moodle. You should check regularly for such materials. Professor Goldys Notes from 2009 will be provided in Week 2 or 3.
Course Evaluation and Development

The School of Mathematics and Statistics evaluates each course each time it is run. We carefully consider the student responses and their implications for course development. It is common practice to discuss informally with students how the course and their mastery of it are progressing.

Administrative matters

Additional Assessment

See the School of Mathematics and Statistics web page at http://www.maths.unsw.edu.au/students/current.

School Rules and Regulations

Fuller details of the general rules regarding attendance, release of marks, special consideration, etc., are available via the School of Mathematics and Statistics Web page at http://www.maths.unsw.edu.au/students/current/policies/studentpolicy.html.

Plagiarism and academic honesty

Plagiarism is the presentation of the thoughts or work of another as one’s own. Issues you must be aware of regarding plagiarism and the university’s policies on academic honesty and plagiarism can be found at http://www.lc.unsw.edu.au/plagiarism and http://www.lc.unsw.edu.au/plagiarism/plagiarism_STUDENTBOOK.pdf.
Detailed course schedule

1. Conditional Expectation
2. Filtrations and Adapted Processes
3. Martingales
4. Standard Brownian Motion
5. Stopping Times and Martingales
6. Itô Stochastic Integral
   6.1. Itô Integral for Elementary Processes
   6.2. Itô Integral for Processes from $L^2_P(W)$
   6.3. Itô Integral for Processes from $L_2(W)$
7. Continuous Local Martingales
8. Continuous Semimartingales and Itô Processes
9. Itô’s Lemma
   9.1. One-dimensional Case
   9.2. Multi-dimensional Case
   9.3. Itô-Tanaka-Meyer Formula
10. Lévy’s Characterization Theorem
11. Martingale Representation Property
12. Stochastic Differential Equations
   12.1. Linear Stochastic Differential Equation
   12.2. Itô’s Existence and Uniqueness Theorem for SDEs
13. Stochastic Exponential
14. Radon-Nikodým Density
15. Girsanov’s Theorem
17. Feynman-Kac Formula
18. Jump processes