

## Mike's Monthly Maths Mysteries Problem 1

On the 4th of December 1903, Srinivasa Ramanujan had to sit the following algebra examination as part (one third) of the maths exam to matriculate to the University of Madras.

He was almost 16 years old.

He had two hours to do as well as he could. How well can you do in two hours?

I'll give you a month, but you must not use a computer or a calculator.

I. 1. Multiply

$$2 \left( \frac{x}{y} + 1 + \frac{y}{x} \right) \text{ by } \frac{y}{2x} - 1 + \frac{x}{2y}.$$

2. Divide  $x^6 - a^3x^3 - 2a^6$  by

$$\frac{x^2}{a^2} - \frac{x}{a} + 1.$$

3. Find the value of

$$\left( \frac{x}{x-a} \right)^2 + \left( \frac{x}{x+a} \right)^2 \text{ in terms of } n \text{ when } x^2 = a^2 \left( \frac{n+1}{n-1} \right).$$

II. Resolve into elementary factors:—

1.  $(a^2b^2 - 1)(x^2 - y^2) + 4abxy;$

2.  $216x^6 + 19x^3 - 1.$

III. Find the H.C.F. of

$$27a^5 - 45a^4 - 16 \quad \text{and} \quad 18a^5 - 45a^4 - 5a - 14.$$

IV. Reduce to their simplest forms the expressions:

1.  $x(y + z - x)^2 + y(z + x - y)^2 + z(x + y - z)^2$   
 $+ (x + y - z)(y + z - x)(z + x - y);$
2.  $\left( \frac{x + 4}{x^2 - x - 12} - \frac{x + 3}{x^2 + x - 12} \right) / \left( 1 + \frac{2(x^2 - 12)}{x^2 + 7x + 12} \right).$

V. Extract the square root of the expression

$$(x^2 + y^2 - 2)^2 + 4(xy + 2)(x^2 + xy + y^2).$$

VI. Solve the equations:—

1.  $\frac{2x^2 - x - 1}{2x - 1} + \frac{6x^2 - 4x + 1}{3x - 2} = \frac{2}{6x - 13} + \frac{6x^2 - 9x - 1}{2x - 3};$
2.  $2\frac{3}{5} + \frac{3x - 5y}{2} = \frac{2}{5}(x + 2), \quad 8 - \frac{x - 2y}{4} = \frac{x}{2} + \frac{y}{3};$
3.  $ab(x^2 + 1) = x(a^2 + b^2).$

VII. B spends 32 Rupees a month, and A, whose monthly income is to B's as 7:5, spends three times as much as B saves. If A's income were increased in the ratio of 7:10 and his expenditure in the ratio of 6:7, he would save 21 Rupees a month more than before. Find B's income.