Modelling Operational Risk

Pavel Shevchenko

CSIRO Division of Mathematical and Information Sciences, Sydney

Quantitative Risk Management group

Sydney, Australia

E-mail: Pavel.Shevchenko@csiro.au

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- **Basel II recognises the importance of the potential impact of losses due to Operational Risk and requires that banks hold adequate capital to protect against these losses.**

- **In Australia, the national regulator (APRA) is now applying the same detailed scrutiny to Operational Risk as previously to credit risk and market risk.**

- **The BCBS (Basel Committee on Banking Supervision) defined Operational Risk as:**

  the risk of loss resulting from inadequate or failed internal processes, people and systems, or from external events

  includes legal, excludes strategic and reputational risks

Under the Basel II framework, banks can estimate operational risk using one of three approaches:

- **The Basic Indicator Approach**
  (15% of Gross income averaged over three year)

- **The Standardised Approach**
  (12-18% on the business line level)

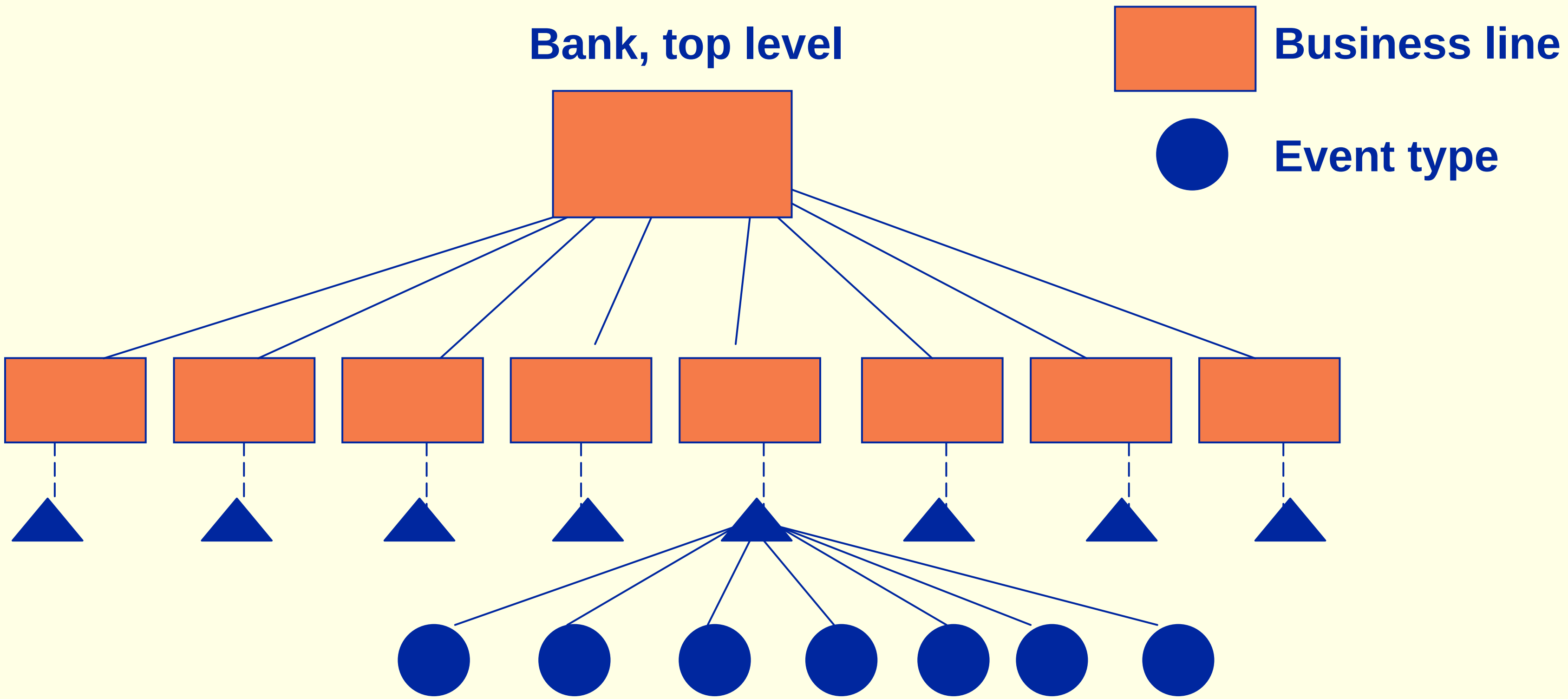
- **The Advanced Measurement Approaches (AMA)**
  Internal model for 56 risk cells (7 event types x 8 business line)

BCS has identified the following 7 risk event types:

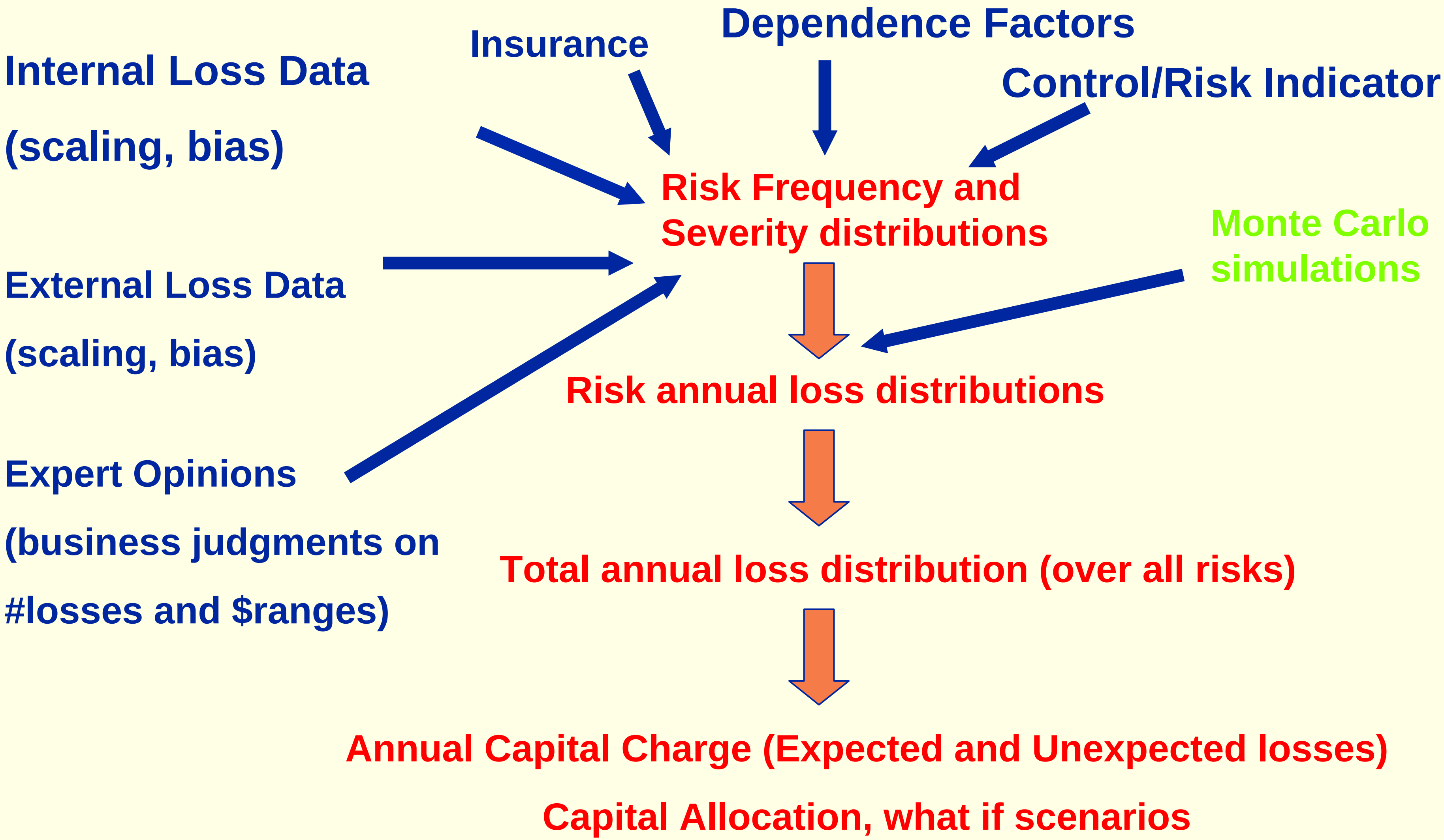
- **Internal fraud:** e.g. intentional misreporting, employee theft, insider trading
- **External fraud:** e.g. robbery, cheque forgery, damage from computer hacking
- **Employment practices and workplace safety:** e.g. workers' compensation claims, violation of employee OH&S rules, union activities, discrimination claims
- **Clients, products and business practices:** e.g. misuse of confidential customer information, improper trading activities, money laundering, sale of unauthorised products.
- **Damage to physical assets:** e.g. terrorism, vandalism, earthquakes, fires, floods.
- **Business disruption and system failures:** e.g. hardware and software failures, telecommunication problems, utility outages, computer viruses.
- **Execution, delivery and process management:** e.g. data entry errors, management failures, incomplete legal documentation, unapproved access to client accounts

8 Business Lines

- Corporate finance(18%)
- Payment&Settlement(18%)
- Trading&Sales(18%)
- Agency Services(15%)
- Retail banking(12%)
- Asset management(12%)
- Commercial banking(15%)
- Retail brokerage(12%)

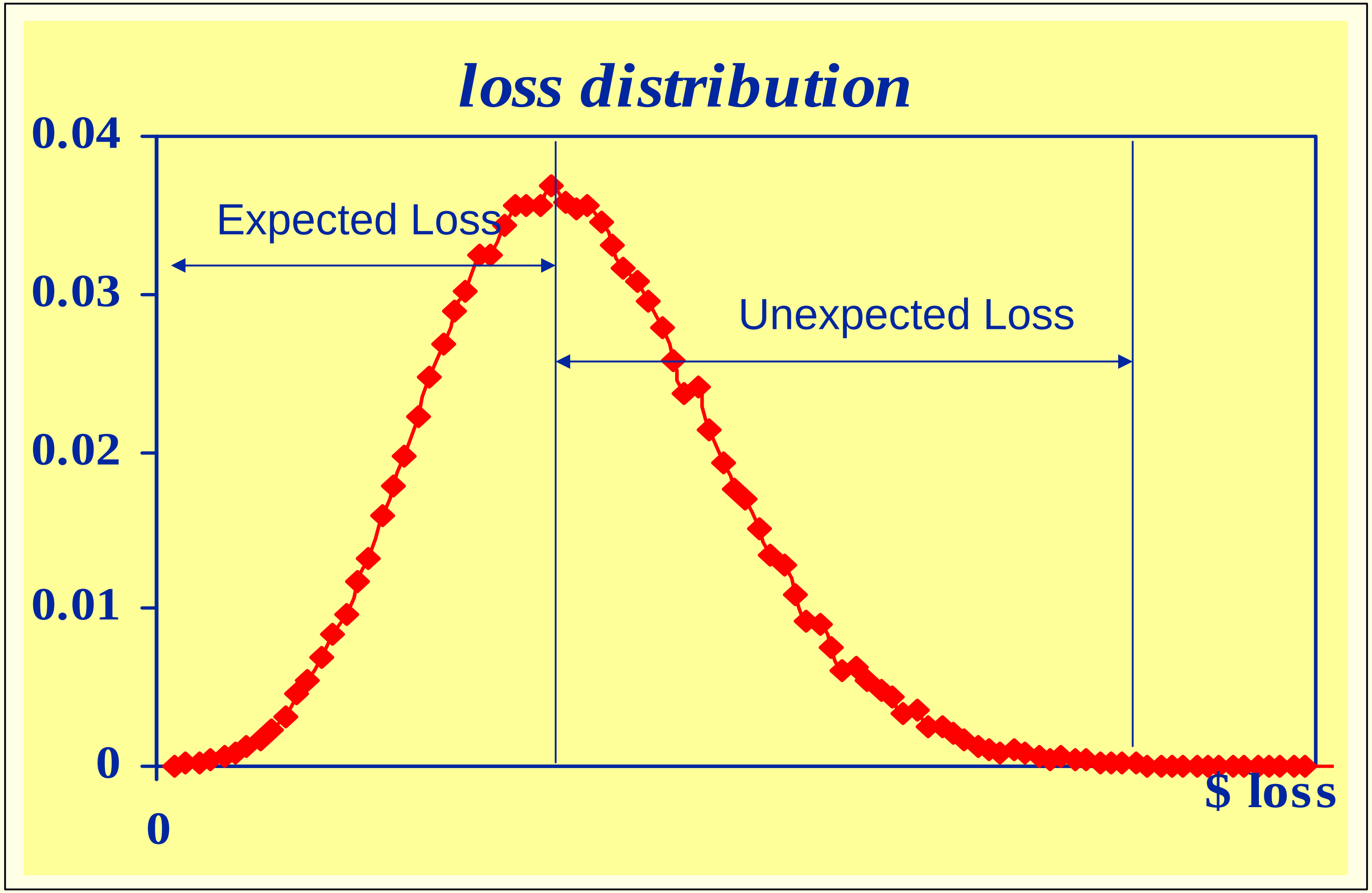

Bank, top level
Business line
Event type

Advanced Measurement Approach: bottom-up Loss Distribution Approach



Annual Capital Charge

unexpected loss=$VaR_{0.999}$-Expected Loss; Pr [Loss≤$VaR_{0.999}$]=0.999



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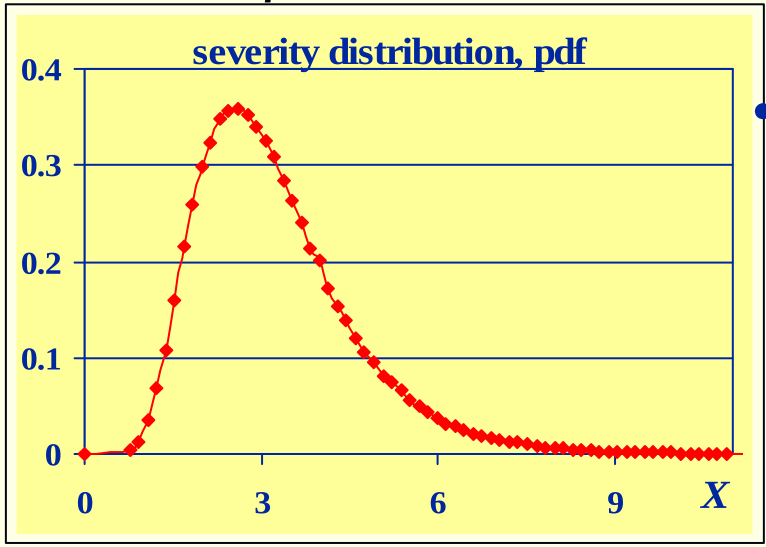
Challenges/Tools

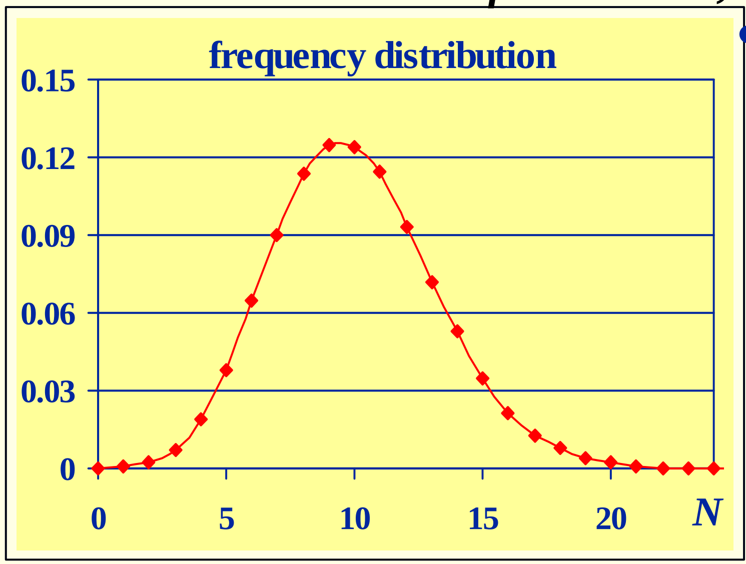
- Definition, identification, measurement, monitoring, indicators/controls
- Data Truncation: known threshold, stochastic threshold, unknown threshold
- Limited Data: mixing internal, external data and expert opinions via credibility theory, Bayesian techniques
- Data sufficiency: capital charge accuracy
- Correlation between risks and its estimation: copula, common shock processes
- Control indicators: regression/factorial analysis
- OR insurance: point processes
- Non-Gaussian distributions, Fat tails: EVT, mixed distributions, splices
- VaR pitfalls: coherent risk measures, expected shortfall
- Capital allocation

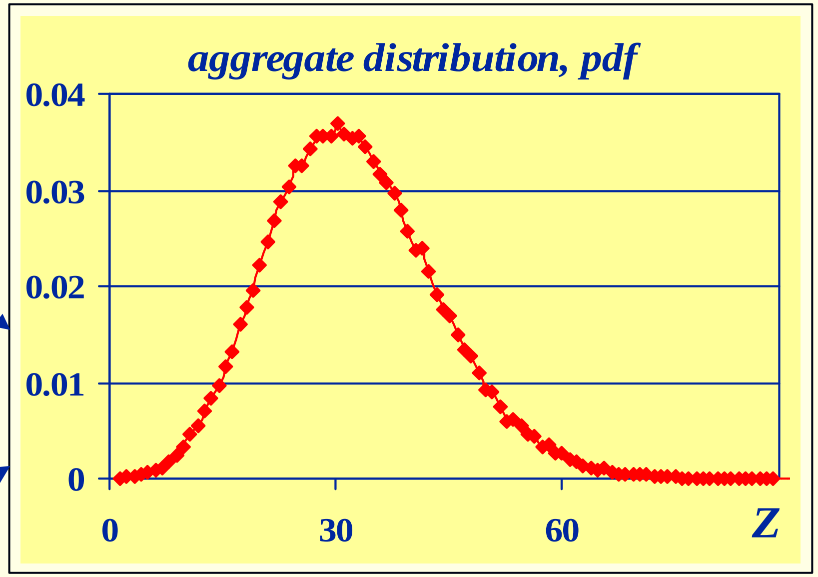
Loss Distribution Approach: single risk cell (business line/event type)

loss of the event, X



annual number of events, N





$$\text{annual loss}, Z = \sum_{k=1}^{N} X_k$$

Monte Carlo, semi-analytic

$N \sim P(.|\boldsymbol{\theta}),$ **e.g. Poisson**

$X_1,\ldots,X_N$ *are iid* $\sim f(.|\xi)$ **e.g. LogNormal**

N *and* $X_1,\ldots,X_N$ *are independent*

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Insurance for Operational Risks

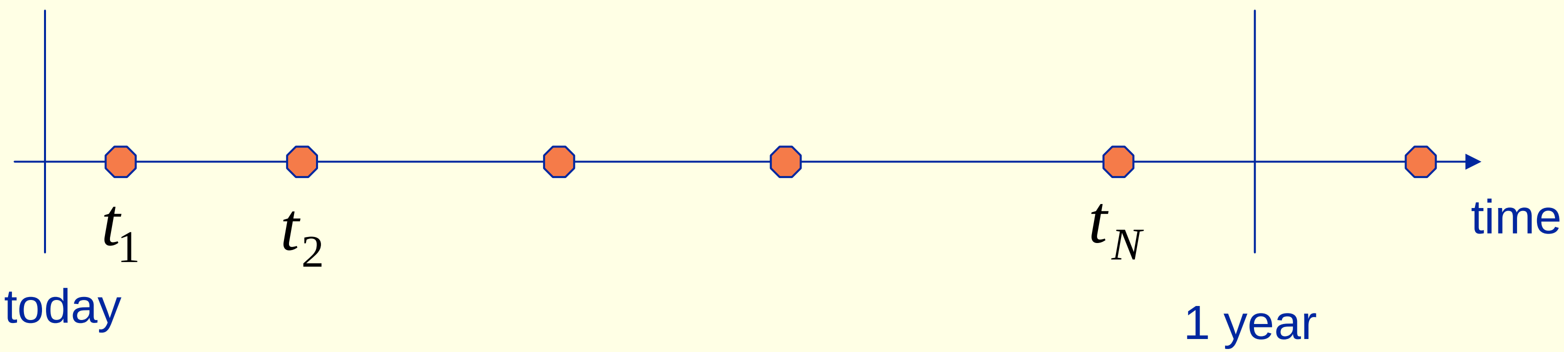
- Insurance: probability of coverage, insurer default, cover limit, excess, regulatory cap
- Modelling of loss event times is required instead of event frequency to address OR insurance.
- point process: $0 < t_1 < \ldots < t_N \leq 1 < t_{N+1} < \ldots$

  e.g. homogeneous Poisson process

$$\delta t = t_{i+1} - t_i \sim \text{Exponential}(\lambda),\ N \sim \text{Poisson}(\lambda)$$

non-homogeneous Poisson: $\lambda(t)$

doubly stochastic Poisson: $\lambda \sim \text{Gamma}(.) \Rightarrow N \Rightarrow \text{NegativeBinomial}(.)$



Data Truncation models

Known constant truncation level $X_i > L, i = 1,2,\ldots.$

Known variable truncation level $X_i > L_i, i = 1,2,\ldots.$

Unknown truncation level

Stochastic truncation level $L \sim g(.)$

Known threshold

- Known constant truncated level, i.e. loss data $X_i \geq L, i = 1,2,....$

  Untruncated severity distribution $f(X), X \geq 0$

  Truncated severity distribution

$$f(X \mid X > L) = \frac{f(X)}{\Pr[X > L]}; \quad X \geq L; \quad \Pr[X > L] = \int_L^{\infty} f(X) dX$$

- Severity pdf fit via e.g. Maximum Likelihood Method

$$\Psi(\alpha_1, ..., \alpha_N) = \prod_{i=1}^{K} \frac{f(X_i)}{\Pr[X > L]}$$

- Frequency adjustment $N^{(true)} \approx N^{obs} / \Pr[X > L]$

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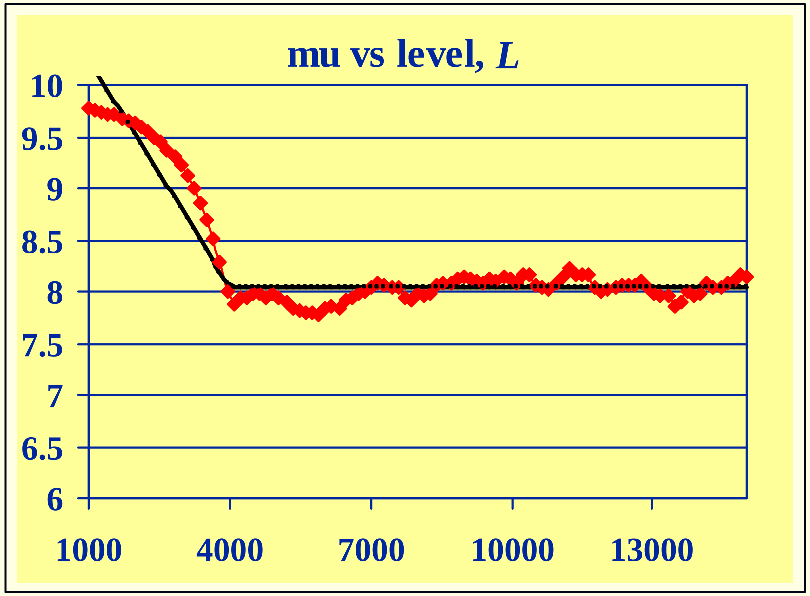
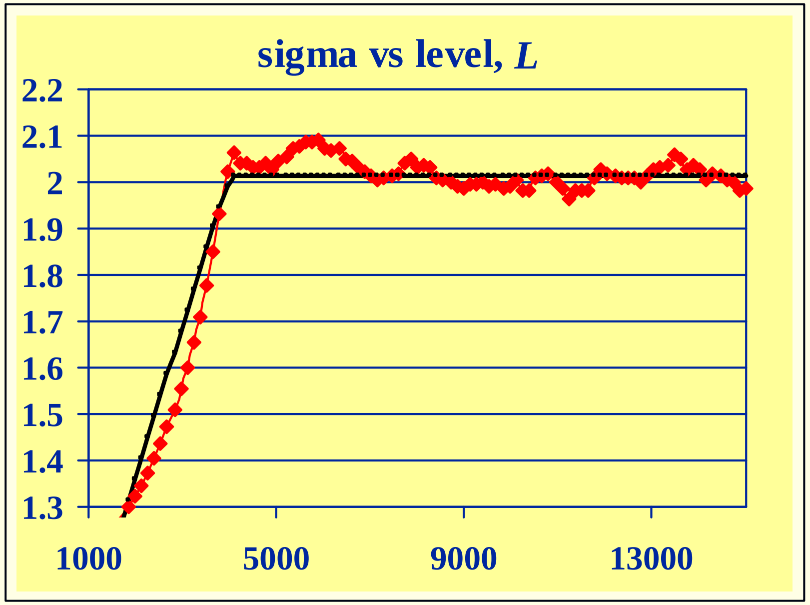
Unknown truncation level

- L is an extra parameter in likelihood function
- L is unknown: $\mu(L) = \begin{cases} \alpha \times L + \beta; & L < \gamma \\ \alpha \times \gamma + \beta; & L \geq \gamma \end{cases}$ $\quad \min_{\alpha,\beta,\gamma} \sum_i [\mu^{obs}(L_i) - \mu(L_i)]^2$

Example: *X*~LogNormal(mu=8, sigma=2), *L*=4000
estimates: mu≈8.05, sigma≈2.01, *L*≈4001





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Stochastic truncation level

Severity distribution of losses $f(X),\ X \geq 0$

reported losses $X_i > L, i = 1, \ldots, K$ ***where*** $L \sim g(.)$

conditional pdf $f(X \mid L = a) = 1_{X>a} \dfrac{f(X)}{\Pr[X > a]}; \quad X \geq a; \quad \Pr[X > a] = \int_a^{\infty} f(y)dy$

marginal distribution of reported losses

$$\tilde{f}(X) = \int_0^{\infty} f(X \mid L = a) g(a) da = f(X) \int_0^{X} \frac{g(a)}{\Pr[X > a]} da \qquad \Psi = \prod_i \tilde{f}(X_i)$$

$$N_{true} = N_{obs} / \Pr[X > L]; \Pr[X > L] = \int_0^{\infty} g(L) dL \int_L^{\infty} f(X) dX$$

Dependence between risks

$$\textit{total annual loss}: Z = \sum_{k=1}^{K} Z_k = \sum_{i=1}^{N_1} X_i^{(1)} + \sum_{i=1}^{N_2} X_i^{(2)} + \ldots + \sum_{i=1}^{N_K} X_i^{(K)}$$

- Diversification: $C(Z=R_1+\ldots+R_n)<C(R_1)+\ldots+C(R_n)$
- VaR: $VaR_\alpha(Z) = F_Z^{-1}(\alpha) = \min\{z, F_Z(z) \geq \alpha\}$
- Conditional VaR (CVaR): $CVaR_\alpha(Z) = E[Z \mid Z > VaR_\alpha(Z)]$
- Dependence between frequencies
- Dependence between event point processes
- Dependence between severities
- Dependence between annual losses
- Dependence between risk profiles (parameters)

$$total\ annual\ loss: Z = \sum_{k=1}^{K} Z_k = \sum_{i=1}^{N_1} X_i^{(1)} + \sum_{i=1}^{N_2} X_i^{(2)} + \ldots + \sum_{i=1}^{N_K} X_i^{(K)}$$

Basel Committee statement*:*

"Risk measures for different operational risk estimates must be added for purposes of calculating the regulatory minimum capital requirement. However, the bank may be permitted to use internally determined correlations in operational risk losses across individual operational risk estimates, provided it can demonstrate to a high degree of confidence and to the satisfaction of the national supervisor that its systems for determining correlations are sound, implemented with integrity, and take into account the uncertainty surrounding any such correlation estimates (particularly in periods of stress). The bank must validate its correlation assumptions."

Adding capitals=>perfect dependence between risks (too conservative)

Dependence between frequencies via copula

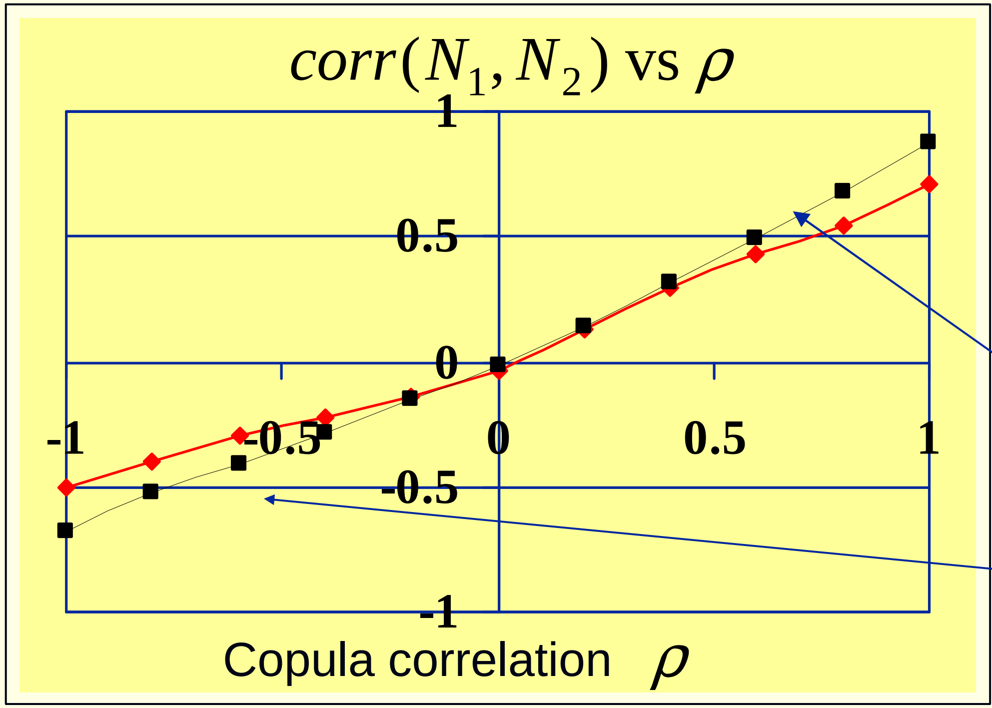
$$C(U_1, U_2, \ldots, U_K)), U_i \sim Uniform(0,1)$$

$$N_1 = F_1^{-1}[U_1], \ldots., N_K = F_K^{-1}[U_K]$$

e.g. Gaussian copula $C_\rho^{Ga}(u_1, \ldots, u_d) = F_N^{(\rho)}(F_N^{-1}(u_1), \ldots, F_N^{-1}(u_K))$

$$corr(N_i, N_j) \neq 0 \; if \;\; \rho \neq 0$$

Example



$$N_1 \sim Poisson(\lambda_1)$$

$$N_2 \sim Poisson(\lambda_2)$$

$$corr(N_1, N_2) \neq \rho$$

■ $\lambda_1 = 0.5, \lambda_2 = 1$

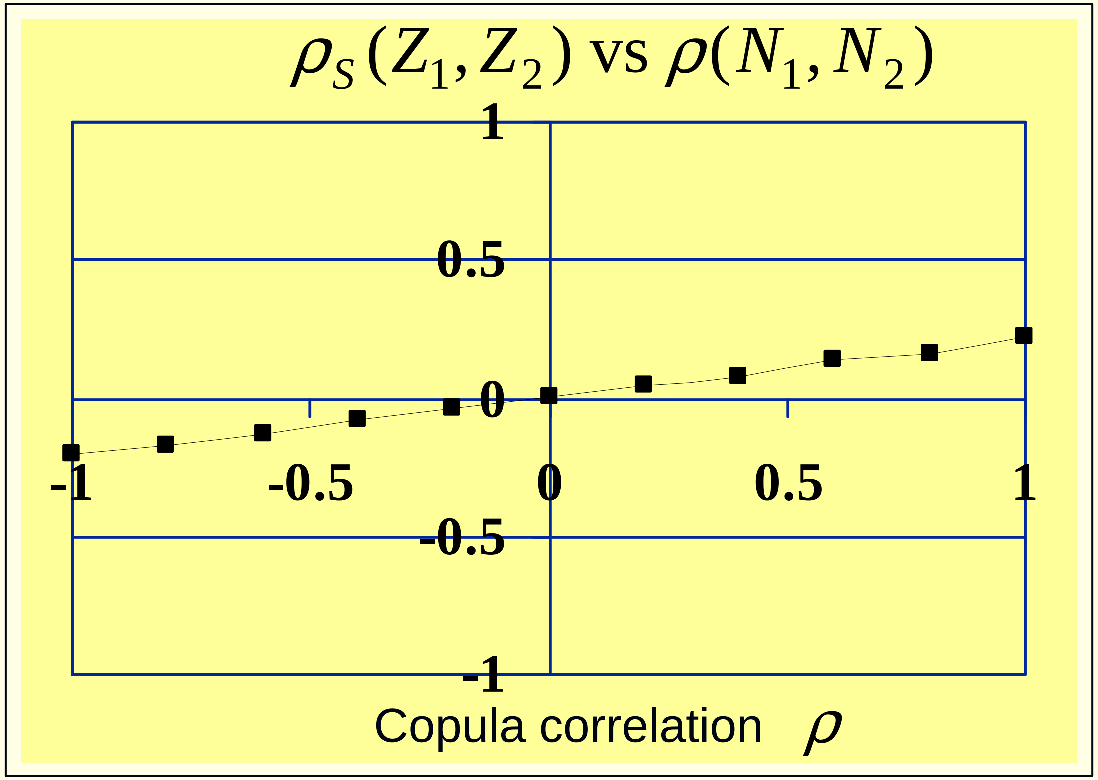
■ $\lambda_1 = 5, \lambda_2 = 10$

Dependence between frequencies=>Dependence between annual losses

Example

$$N_1 = F^{-1}_{Poisson}[U_1], N_2 = F^{-1}_{Poisson}[U_2] \qquad C(U_1,U_2) = C^{Ga}_{\rho}(U_1,U_2)$$

$$Z_1 = \sum_{i=1}^{N_1} X_i^{(1)} \quad Z_2 = \sum_{i=1}^{N_2} X_i^{(2)} \quad X^{(1)} \sim \text{LogNormal}(1,2), X^{(2)} \sim \text{LogNormal}(1,2), X^{(1)} \text{ ind } X^{(2)}$$



$$N_1 \sim Poisson(\lambda_1)$$

$$N_2 \sim Poisson(\lambda_2)$$

$$\lambda_1 = 5, \lambda_2 = 10$$

Dependence via common Poisson process

(Johnson, Kotz and Balakrishnan)

$$N_1(t) \sim \hat{N}_1(t) + N_C(t)$$

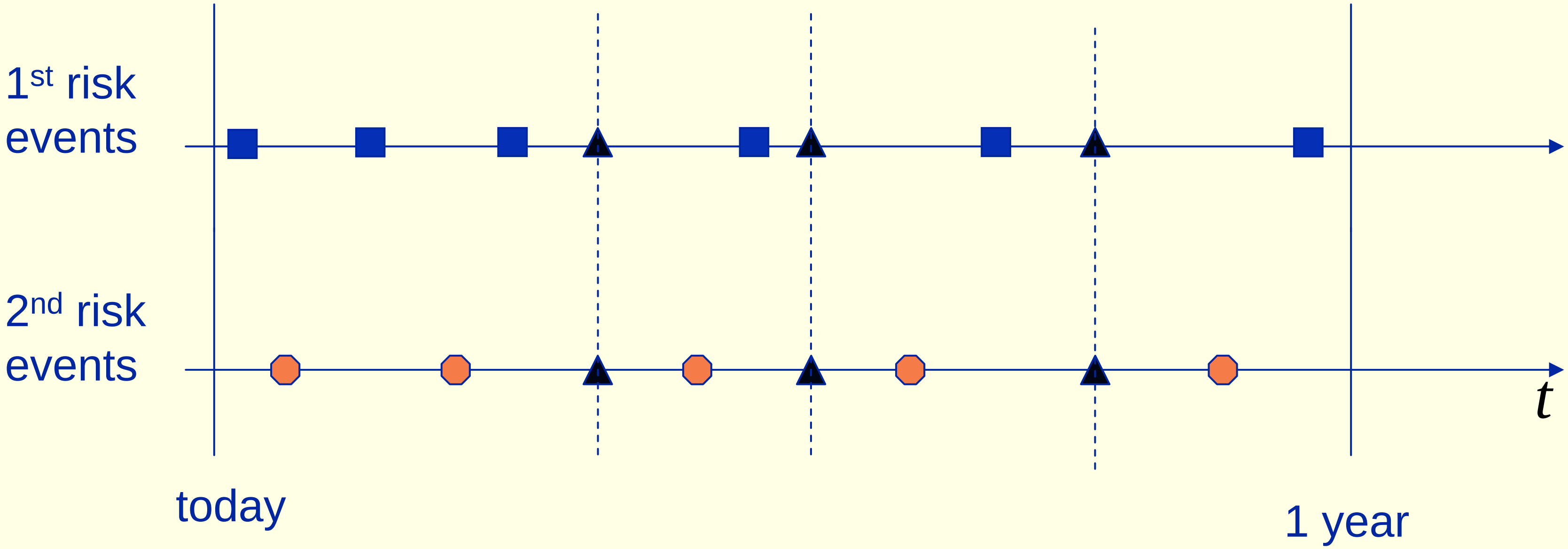
$$N_2(t) \sim \hat{N}_2(t) + N_C(t)$$

$$\hat{N}_1(t) \sim Poisson(\lambda_1); \hat{N}_2(t) \sim Poisson(\lambda_2); N_C(t) \sim Poisson(\lambda_C)$$

$$N_1(t) \sim Poisson(\lambda_1 + \lambda_C); N_2(t) \sim Poisson(\lambda_2 + \lambda_C); corr(N_1, N_2) = \lambda_C / \sqrt{(\lambda_1 + \lambda_C)(\lambda_2 + \lambda_C)}$$

positive dependence; constant covariance

extension $N_i(t) = \begin{cases} \hat{N}_i(t) + N_C(t) & \text{with prob } p_i \\ \hat{N}_i(t) & \text{with prob } 1 - p_i \end{cases}$ $\quad \mathrm{cov}(N_i, N_k) = \lambda_C p_i p_k$



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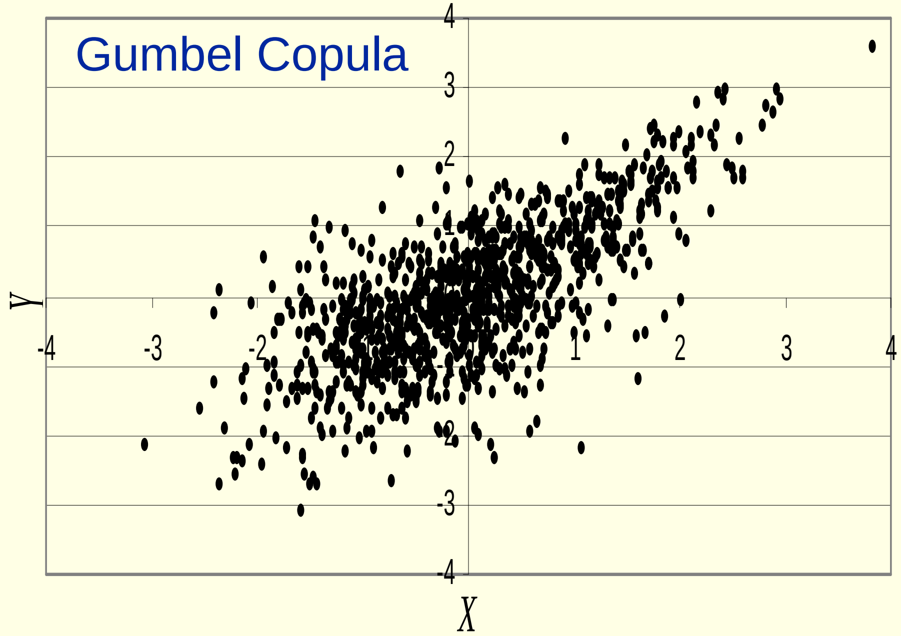
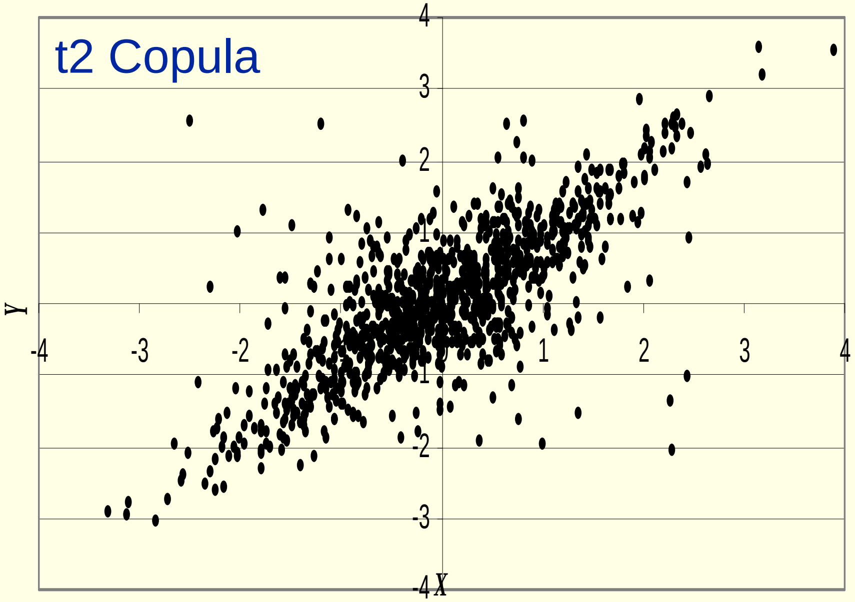
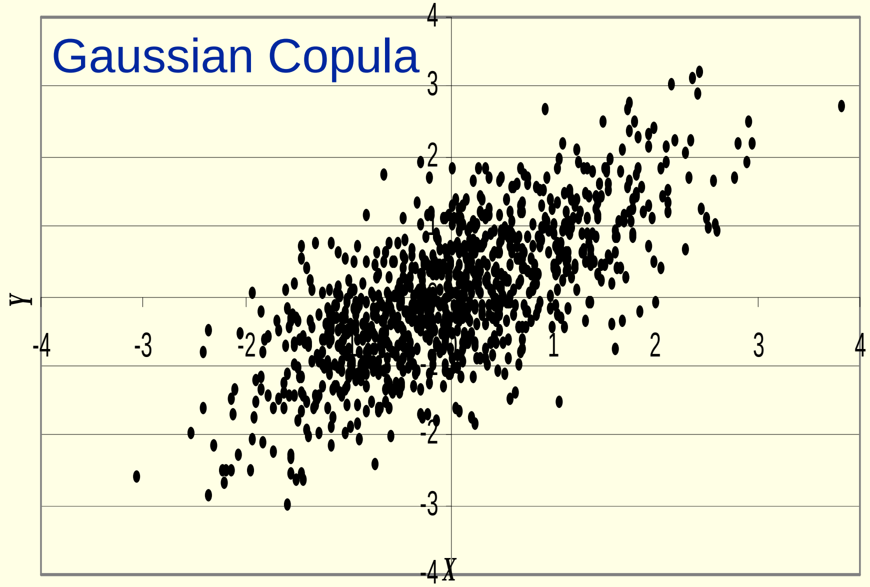
Dependence via copula



$$X \sim Normal(0,1); Y \sim Normal(0,1)$$

$$corr(X,Y) = 0.7$$





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Upper tail dependence

$$\lambda = \lim_{\alpha \to 1-} P[Y > F_2^{-1}(\alpha) \mid X > F_1^{-1}(\alpha)] = \lim_{\alpha \to 1-} \frac{1 - 2\alpha + C(\alpha,\alpha)}{1-\alpha}$$

***Gaussian* copula** $\lambda = 0$

$$C_\rho^{Ga}(u_1, u_2, \ldots, u_d) = F_N^{(\rho)}(F_N^{-1}(u_1), F_N^{-1}(u_2), \ldots, F_N^{-1}(u_d))$$

***t*-copula** $\lambda \geq 0$

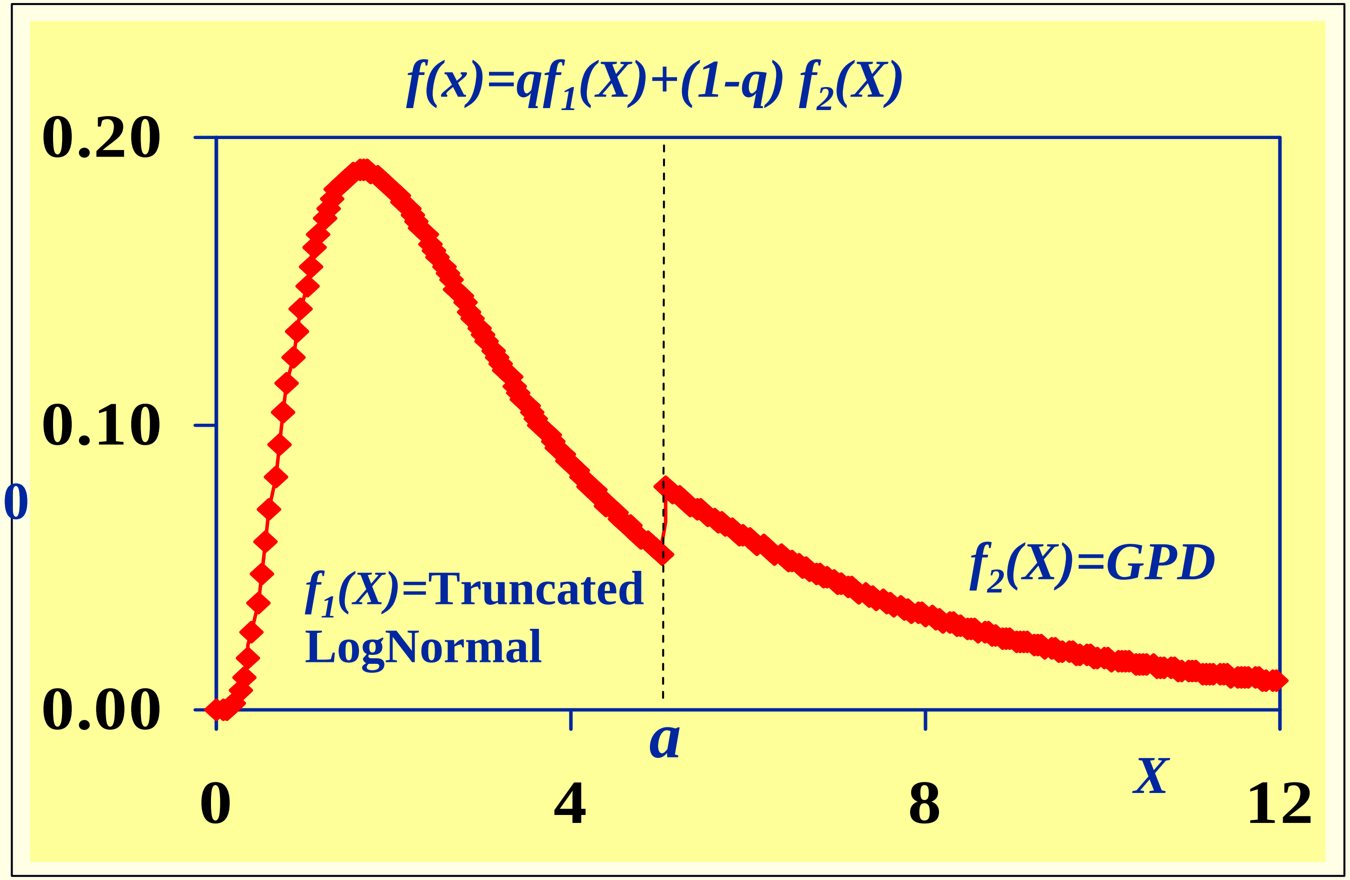
$$C_\Sigma^{t\nu}(u_1, u_2, \ldots, u_d) = F_{t_\nu}^{(\Sigma)}(t_\nu^{-1}(u_1), t_\nu^{-1}(u_2), \ldots, t_\nu^{-1}(u_d))$$

***Gumble* copula** $\lambda = 2 - 2^\beta$

$$C_\beta^{Gu}(u_1, \ldots, u_d) = \exp\left\{ -\left[(-\ln u_1)^{1/\beta} + \ldots + (-\ln u_d)^{1/\beta} \right]^\beta \right\}, \quad 0 < \beta \leq 1$$

Severity Distribution Tail: Extreme value Theory and splicing

EVT Generalized Pareto Distribution (GPD) $H(x) = \begin{cases} 1-(1+\xi x/\beta)^{-1/\xi}; & \xi \neq 0 \\ 1-\exp[-x/\beta]; & \xi = 0 \end{cases}$



Mixture distributions: $f(X) = w_1 f_1(X) + \ldots + w_K f_K(X), \quad f_i(X), 0 \leq X < \infty$

Structural Modelling of Operatinal Risk via Bayesian Inference: combining loss data with expert opinions

Pavel Shevchenko (CSIRO) and Mario Wüthrich (ETH)

Structural Modelling of Operational Risk using Bayesian Inference: combining loss data with expert opinions.

June 2006. Submitted to The Journal of Operational Risk

Bayesian inference to combine expert opinions with loss data

$$h(\vec{X},\vec{\alpha}) = h(\vec{X} \mid \vec{\alpha})\pi(\vec{\alpha}) = \hat{\pi}(\vec{\alpha} \mid \vec{X})h(\vec{X})$$

- Expert opinions for prior distribution $\pi(\vec{\alpha})$
- Loss data $\vec{X} = \{X_1,..., X_n\}; \quad h(\vec{X} \mid \vec{\alpha})$
- Posterior distribution $\hat{\pi}(\alpha \mid \vec{X}) = h(\vec{X} \mid \vec{\alpha})\pi(\vec{\alpha}) / h(\vec{X})$

Poisson-Gamma

$\mathbf{N} = (N_1, \ldots, N_n)$ are conditionally iid from $f(N \mid \lambda) = e^{-\lambda} \frac{\lambda^N}{N!}, \quad \lambda \geq 0$

$$\pi(\lambda \mid \alpha, \beta) = \frac{(\lambda/\beta)^{\alpha-1}}{\Gamma(\alpha)\beta} \exp(-\lambda/\beta), \quad \lambda > 0, \alpha > 0, \beta > 0$$

$$\hat{\pi}(\lambda \mid \mathbf{N}) \propto \frac{(\lambda/\beta)^{\alpha-1}}{\Gamma(\alpha)\beta} \exp(-\lambda/\beta) \prod_{i=1}^{n} e^{-\lambda} \frac{\lambda^{N_i}}{N_i!} \propto \lambda^{\hat{\alpha}-1} \exp(-\lambda/\hat{\beta})$$

$$\alpha \rightarrow \hat{\alpha} = \alpha + \sum_{i=1}^{n} N_i,$$

$$\beta \rightarrow \hat{\beta} = \beta/(1 + \beta \times n).$$

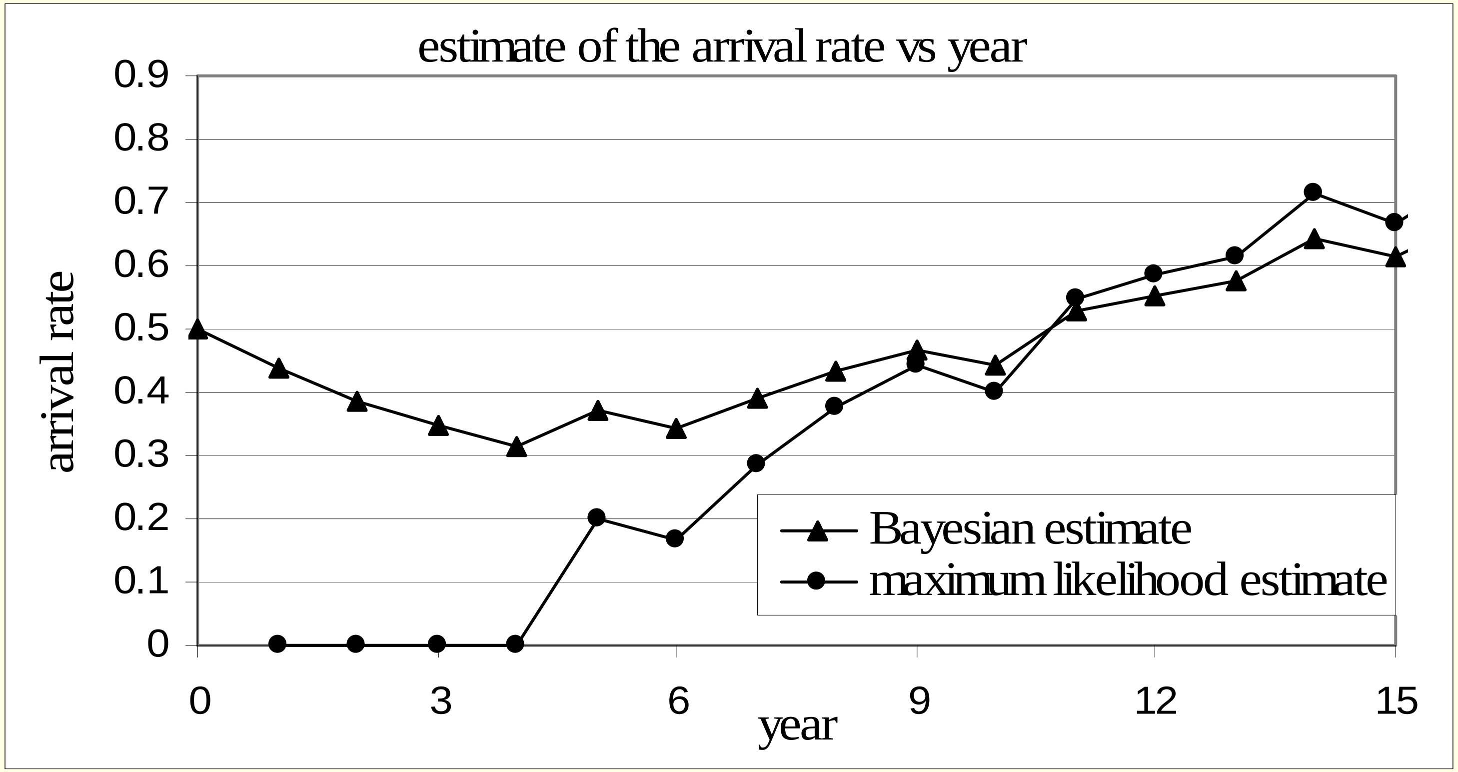
Example

Annual counts **N**=(0, 0, 0, 0, 1, 0, 1, 1, 1, 0, 2, 1, 1, 2, 0) from Poisson $\lambda = 0.6$

expert opinions $E[\lambda] = 0.5, \Pr[0.25 \le \lambda \le 0.75] = 2/3 \to \alpha \approx 3.41, \beta \approx 0.15$

$\hat{\lambda}_k = \hat{\alpha}_k \times \hat{\beta}_k$ the Bayesian estimator with Gamma prior $\alpha \approx 3.41, \beta \approx 0.15$

$\tilde{\lambda}_k = \frac{1}{k}\sum_{i=1}^{k} N_i$ the Maximum Likelihood estimator

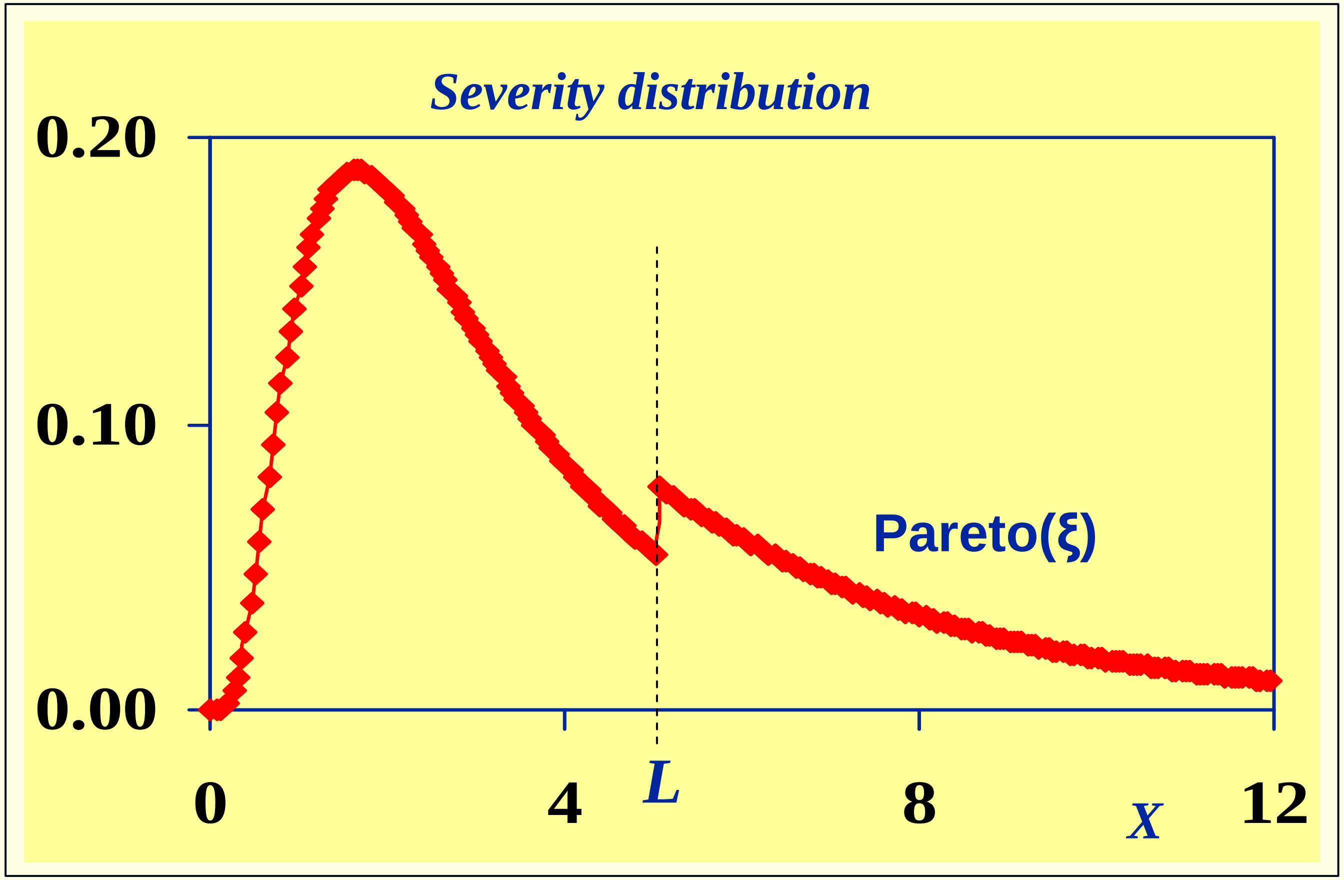


“Toy” Model for Operational Risk using Credibility Theory

Hans Bühlmann (ETH), Pavel Shevchenko (CSIRO) and Mario Wüthrich (ETH)
A *“Toy” Model for Operational Risk Quantification using Credibility Theory.*
June 2006. Submitted to Risk Magazine

Low Frequency High Impact Losses: Poisson-Pareto

$$f(x|\xi)=\frac{\xi}{L}\left(\frac{x}{L}\right)^{-\xi-1}, x\geq L, \xi>0; \quad P(n|\theta)=\frac{\theta^{n}}{n!}e^{-\theta}, \; n=0,1,\ldots, \; \theta\geq 0$$



Bühlmann - Straub model : consider a portfolio of J risks with observations $Y_{j,k} : k = 1,\ldots,K_j$. Assume that for known weights $w_{j,k}$, θ_j (realization of rv Θ_j) is a risk profile of the j-th risk and

$a)$ $Y_{j,k} : k = 1,\ldots,K_j$ are conditionally independent with

$$E[Y_{j,k} \mid \Theta_j] = \mu(\Theta_j),\ \mathrm{var}[Y_{j,k} \mid \Theta_j] = \sigma^2(\Theta_j)/w_{j,k}$$

$b)$ $(\Theta_1, \mathbf{Y}_1),\ldots,(\Theta_J, \mathbf{Y}_J)$ are independent

$c)$ $\Theta_1,\ldots,\Theta_J$ *are iid*

Define : $\mu_0 = E[\mu(\Theta_j)],\ E[\sigma^2(\Theta_j)] = \sigma^2,\ \mathrm{var}[\mu(\Theta_j)] = \tau^2$

Then homogeneous credibility estimator is

$$\hat{\mu}(\Theta_j) = \alpha_j Y_j + (1-\alpha_j)\hat{\mu}_0, \quad \hat{\mu}_0 = \sum_{j=1}^{J} \frac{\alpha_j}{\alpha_0} Y_j,\ Y_j = \sum_{k=1}^{K_j} \frac{w_{j,k}}{\tilde{w}_j} Y_{j,k},$$

$$\alpha_j = \frac{\tilde{w}_j}{\tilde{w}_j + \sigma^2/\tau^2},\ \alpha_0 = \sum_{j=1}^{J} \alpha_j,\ \tilde{w}_j = \sum_{k=1}^{K_j} w_{j,k}$$

Maximum Likelihood Estimator for tail parameter:

Consider risk cells $j = 1, \ldots, J$ with losses $X_{j,k} \geq L,\ k = 1, \ldots, K_j$

$X_{j,k}$ are (conditionally) iid from Pareto$(\xi_j = a_j \vartheta_j)$

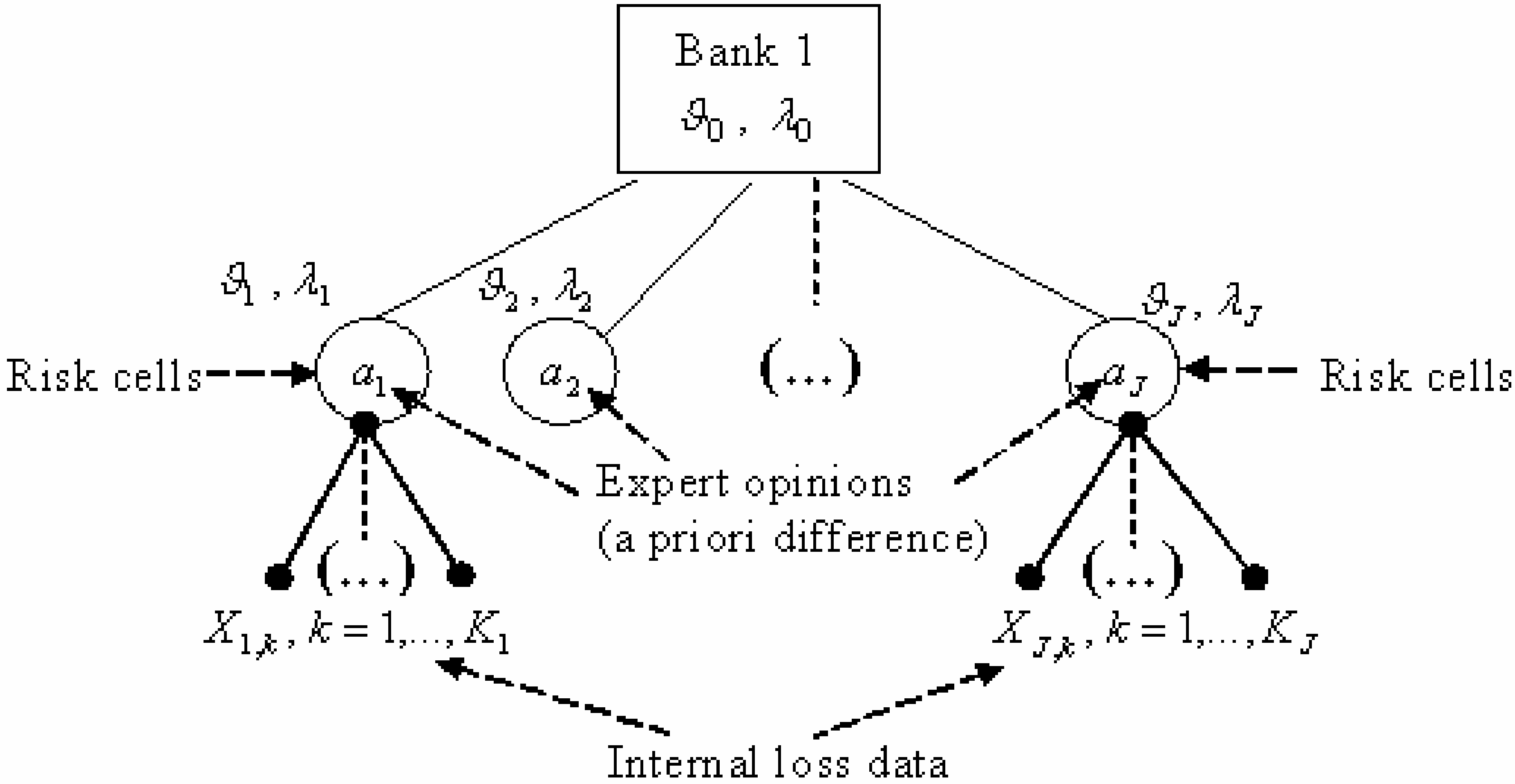
Then the "maximum likelihood estimator"

$$\hat{\vartheta}_j = \left[\frac{a_j}{K_j - 1} \sum_{k=1}^{K_j} \ln\left(\frac{X_{j,k}}{L} \right) \right]^{-1},$$

$$E[\hat{\vartheta}_j \mid \vartheta_j] = \vartheta_j,\ Var[\hat{\vartheta}_j \mid \vartheta_j] = \frac{\vartheta_j^2}{K_j - 2},$$

$$\hat{\xi}_j = a_j \hat{\vartheta}_j$$

Improved credibility estimator (using all data in the bank):



Improved credibility estimator (using all data in the bank):

Assume $\vartheta_j,\ j=1,\dots,J$ are iid with $\mathrm{E}[\vartheta_j]=\vartheta_0,\ Var[\vartheta_j]=\tau_0^2$

ϑ_0 is a risk profile of the bank. Then

$$\hat{\hat{\vartheta}}_j=\alpha_j\hat{\vartheta}_j+(1-\alpha_j)\vartheta_0,\ \text{where}\qquad \alpha_j=\frac{K_j-2}{K_j-1+(\vartheta_0/\tau_0)^2}$$

$$\hat{\tau}_0^2=\frac{1}{J-1}\sum_{j=1}^{J}\alpha_j(\hat{\vartheta}_j-\hat{\vartheta}_0)^2,$$

$$\hat{\vartheta}_0=\frac{1}{W}\sum_{j=1}^{J}\alpha_j\hat{\vartheta}_j,\quad W=\sum_{j=1}^{J^{(1)}}\alpha_j$$

$$\hat{\hat{\xi}}_j=a_j\hat{\hat{\vartheta}}_j,\ j=1,\dots,J$$

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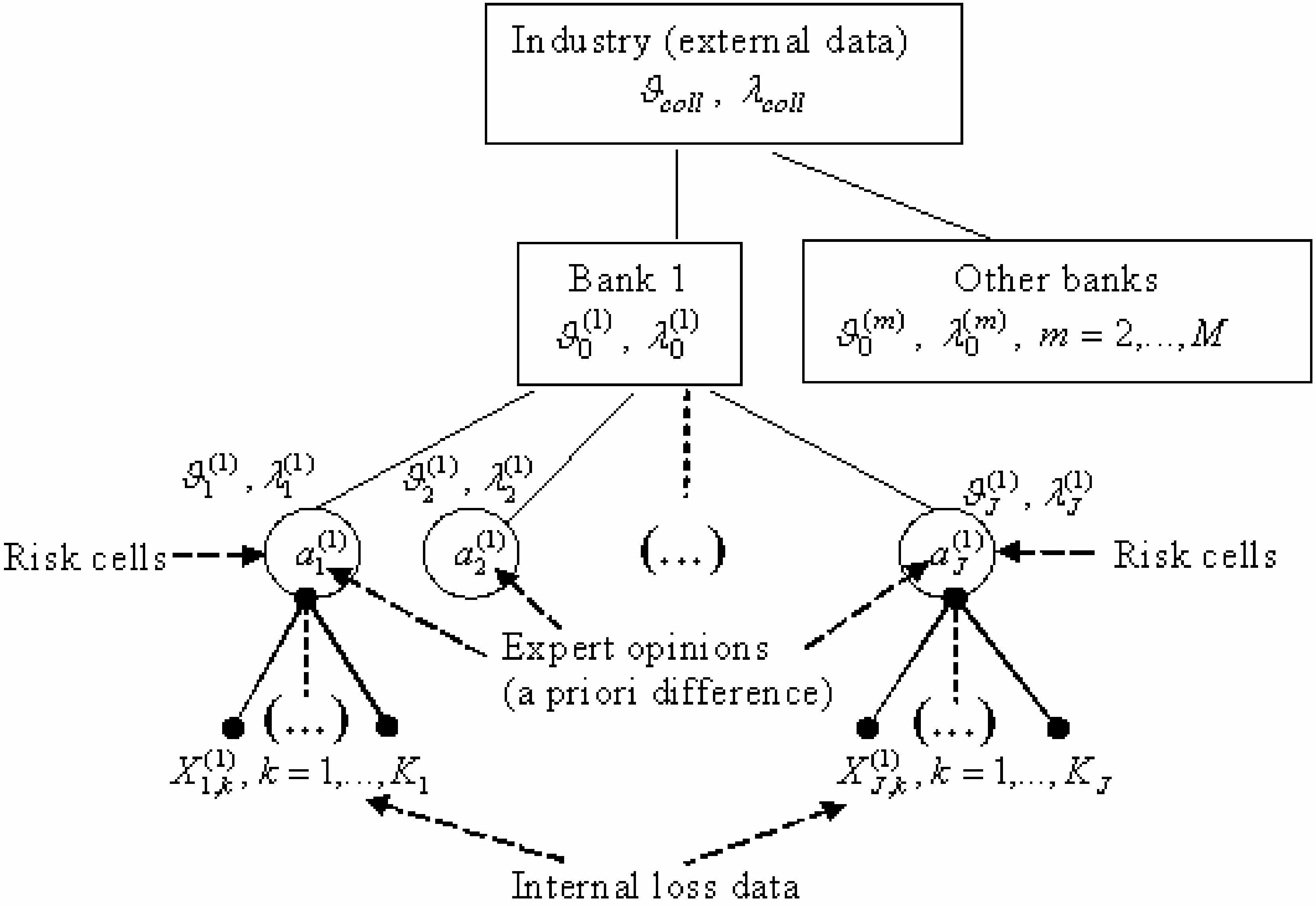
Improved credibility estimator (using industry data):



Improved credibility estimator (using industry data):

Consider M banks with risk profiles $\vartheta_0^{(m)}, m = 1,\ldots,M$

Assume that $\vartheta_0^{(m)}, m = 1,\ldots,M$ are iid with $\mathrm{E}[\vartheta_0^{(m)}] = \vartheta_{\mathrm{coll}}, Var[\vartheta_0^{(m)}] = \tau_{coll}^2$

$\vartheta_{\mathrm{coll}}$ is a risk profile of the industry. Then

$$\hat{\vartheta}_j^{(m)} = \left[\frac{a_j^{(m)}}{K_j^{(m)} - 1} \sum_{k=1}^{K_j^{(m)}} \ln\left(\frac{X_{j,k}^{(m)}}{L^{(m)}} \right) \right]^{-1}, \quad \alpha_j^{(m)} = \frac{K_j^{(m)} - 2}{K_j^{(m)} - 1 + \left(\frac{\vartheta_0^{(m)}}{\tau_0^{(m)}} \right)^2}, \quad \left(\begin{matrix} j = 1,\ldots,J^{(m)} \\ m = 1,\ldots,M \end{matrix} \right)$$

$$\hat{\vartheta}_0^{(m)} = \tfrac{1}{W^{(m)}} \sum_{j=1}^{J^{(m)}} \alpha_j^{(m)} \hat{\vartheta}_j^{(m)}, \quad \beta^{(m)} = \frac{W^{(m)}}{W^{(m)} + \left(\frac{\tau_0^{(m)}}{\tau_{coll}} \right)^2}, \quad W^{(m)} = \sum_{j=1}^{J^{(m)}} \alpha_j^{(m)}, m = 1,\ldots,M$$

$$\hat{\vartheta}_{coll} = \tfrac{1}{A} \sum_{j=1}^{J^{(m)}} \beta^{(m)} \hat{\vartheta}_0^{(m)}, \quad A = \sum_{m=1}^{M} \beta^{(m)}$$

Corrected credibility estimators top - down

$$\hat{\hat{\vartheta}}_0^{(1)} = \beta^{(1)}\hat{\vartheta}_0^{(1)} + (1-\beta^{(1)})\hat{\vartheta}_{coll},$$

$$\hat{\hat{\vartheta}}_j^{(1)} = \alpha_j^{(1)}\hat{\vartheta}_j^{(1)} + (1-\alpha_j^{(1)})\hat{\hat{\vartheta}}_0^{(1)},\ j=1,\ldots,J^{(m)} \Rightarrow \hat{\hat{\xi}}_j^{(1)} = a_j^{(1)}\hat{\hat{\vartheta}}_j^{(1)}$$

Industry structural parameters

$$\hat{\tau}_{coll}^2 = \max\left[c\times\left\{\frac{M}{M-1}\sum_{m=1}^{M}\frac{W^{(m)}}{W_0}(\hat{\vartheta}_0^{(m)} - \overline{\hat{\vartheta}_0^{(m)}})^2 - \frac{M\hat{\tau}^2}{W_0}\right\}, 0\right]$$

$$\hat{\tau}^2 = \frac{1}{M}\sum_{m=1}^{M}(\hat{\tau}_0^{(m)})^2, W_0 = \sum_{m=1}^{M}W^{(m)}, \overline{\hat{\vartheta}_0^{(m)}} = \frac{1}{M}\sum_{m=1}^{M}\hat{\vartheta}_0^{(m)},$$

$$c = \frac{M-1}{M}\left\{\sum_{m=1}^{M}\frac{W^{(m)}}{W_0}\left(1-\frac{W^{(m)}}{W_0}\right)\right\}^{-1}$$

Maximum Likelihood Estimator for arrival rate:

Consider risk cells $j = 1,\ldots,J$ with loss frequencies $N_{j,k}, k = 1,\ldots,K_j$

$N_{j,k}$ are (conditionally) iid from Poisson($\theta_j = \nu_j \lambda_j$)

ν_j are known constants and λ_j are risk profiles

Then the maximum likelihood estimator

$$\hat{\lambda}_j = \frac{1}{\tilde{\nu}_j} \sum_{k=1}^{K_j} N_{j,k}, \quad \tilde{\nu}_j = \nu_j K_j$$

$$E[\hat{\lambda}_j \mid \lambda_j] = \lambda_j, \quad \mathrm{Var}[\hat{\lambda}_j \mid \lambda_j] = \lambda_j / \tilde{\nu}_j,$$

Improved credibility estimator for arrival rate (using bank data)

Assume $\lambda_{\mathrm{j}},\ j=1,\dots,J$ are iid with $\mathrm{E}[\lambda_{\mathrm{j}}]=\lambda_0,\ Var[\lambda_{\mathrm{j}}]=\omega_0^2$

λ_0 is a risk profile of the bank.

Consider $F_{j,k}=N_{j,k}/\nu_j,\ E[F_{j,k}]=\lambda_j,\ \mathrm{Var}[F_{j,k}]=\lambda_j/\nu_j$ then

$$\hat{\hat{\lambda}}_j=\gamma_j\hat{\lambda}_j+(1-\gamma_j)\lambda_0,\ \text{where } \gamma_j=\frac{\tilde{\nu}_j}{\tilde{\nu}_j+\lambda_0/\omega_0^2}\quad\Rightarrow\hat{\hat{\theta}}_j=\nu_j\hat{\hat{\lambda}}_j$$

$$\hat{\omega}_0^2=\max\left[c\times\left\{T-\frac{J\ \hat{\lambda}_0}{\nu_0}\right\},0\right];\quad \hat{\lambda}_0=\frac{1}{\tilde{\gamma}}\sum_j\gamma_j\hat{\lambda}_j,\tilde{\nu}_j=\nu_jK_j$$

$$\nu_0=\sum_{j=1}^{J}\tilde{\nu}_j;\ T=\frac{J}{J-1}\sum_{j=1}^{J}\frac{\tilde{\nu}_j}{\nu_0}(\hat{\lambda}_j-\overline{F})^2;\ \tilde{\gamma}=\sum_j\gamma_j;$$

$$\overline{F}=\frac{1}{J}\sum_{j=1}^{J}\hat{\lambda}_j;\ c=\frac{J}{J-1}\left\{\sum_{j=1}^{J}\frac{\tilde{\nu}_j}{\nu_0}\left(1-\frac{\tilde{\nu}_j}{\nu_0}\right)\right\}^{-1}.$$

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Improved credibility estimator of arrival rate (using industry data)

Consider M banks with risk profiles $\lambda_0^{(m)}, m = 1,\dots,M$

Assume that $\lambda_0^{(m)}, m = 1,\dots,M$ are iid with $\mathrm{E}[\lambda_0^{(m)}] = \lambda_{\mathrm{coll}}, Var[\lambda_0^{(m)}] = \omega_{coll}^2$

λ_{coll} is a risk profile of the industry. Then

$$\hat{\lambda}_j^{(m)} = \frac{1}{\tilde{\nu}_j^{(m)}} \sum_{k=1}^{K_j^{(m)}} N_{j,k}^{(m)}, \quad \gamma_j^{(m)} = \frac{\tilde{\nu}_j^{(m)}}{\tilde{\nu}_j^{(m)} + \lambda_0^{(m)} / (\omega_0^{(m)})^2}, \; j = 1,\dots, J^{(m)}, m = 1,\dots,M$$

$$\hat{\lambda}_0^{(m)} = \frac{1}{W^{(m)}} \sum_{j=1}^{J^{(m)}} \gamma_j^{(m)} \hat{\lambda}_j^{(m)}, \; \rho^{(m)} = \frac{W^{(m)}}{W^{(m)} + \left(\frac{\omega_0^{(m)}}{\omega_{coll}} \right)^2}, \; W^{(m)} = \sum_{j=1}^{J^{(m)}} \gamma_j^{(m)}, m = 1,\dots,M$$

$$\hat{\lambda}_{coll} = \frac{1}{A} \sum_{j=1}^{J^{(m)}} \rho^{(m)} \hat{\lambda}_0^{(m)}, \quad A = \sum_{m=1}^{M} \rho^{(m)}$$

Corrected credibility estimators for arrival rate: top-down

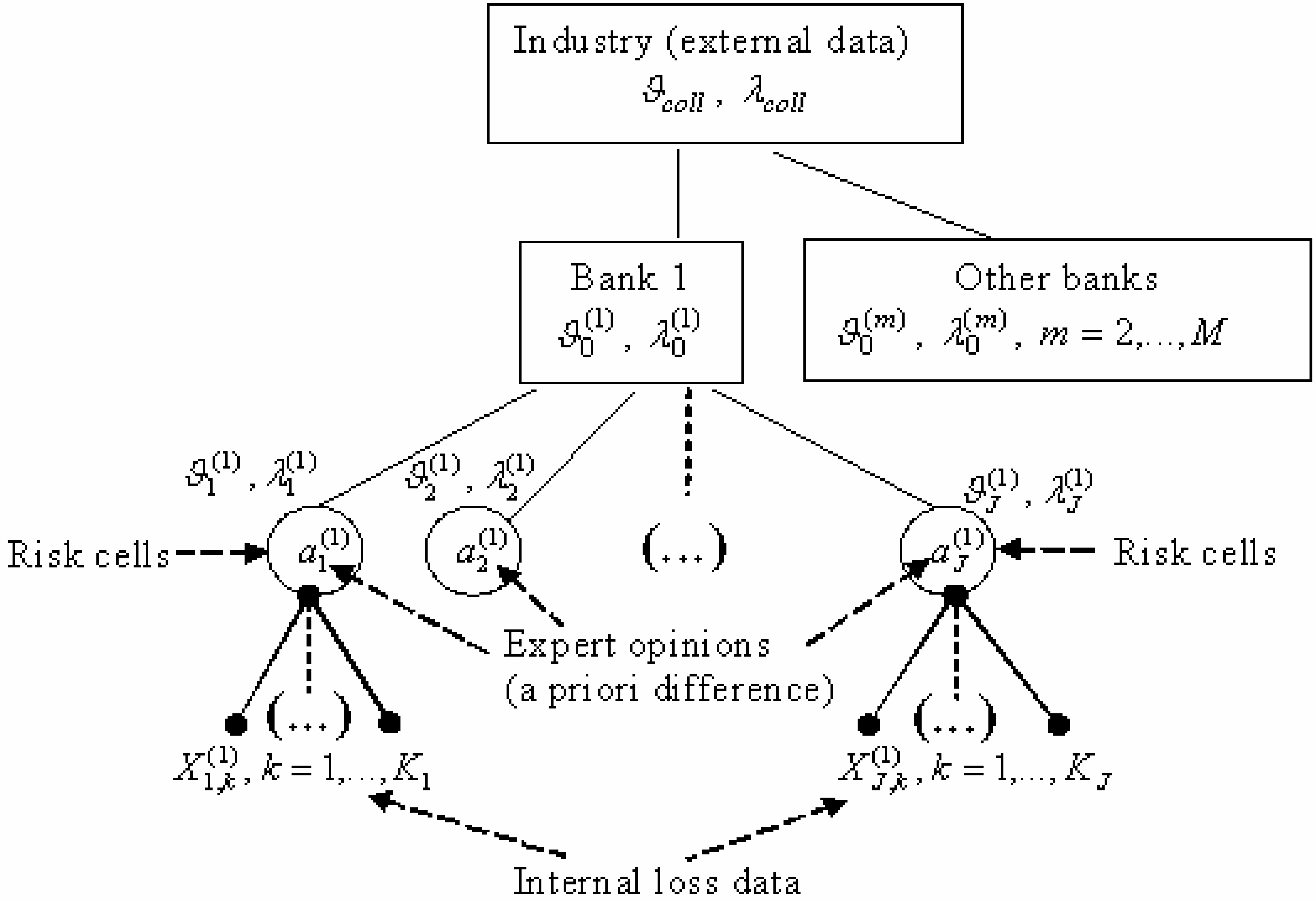
$$\hat{\hat{\lambda}}_0^{(1)} = \rho^{(1)}\hat{\lambda}_0^{(1)} + (1-\rho^{(1)})\hat{\lambda}_{coll},$$

$$\hat{\hat{\lambda}}_j^{(1)} = \gamma_j^{(1)}\hat{\lambda}_j^{(1)} + (1-\gamma_j^{(1)})\hat{\hat{\lambda}}_0^{(1)}, \; j = 1,\ldots, J^{(m)}$$

$$\Downarrow$$

$$\hat{\hat{\theta}}_j^{(1)} = \nu_j^{(1)}\hat{\hat{\lambda}}_j^{(1)}$$

$$Z_j = \sum_{n=1}^{N_j} X_{j,n} \Leftarrow X_{j,n} \sim \text{Pareto}(\hat{\hat{\xi}}_j = a_j \hat{\hat{\vartheta}}_j), \quad N_j \sim \text{Poisson}(\hat{\hat{\theta}}_j = \nu_j \hat{\hat{\lambda}}_j)$$



Topics for further research

- Evolutionary models (stochastic risk profiles)
- Dependence between risks via dependence between risk profiles
- Full Bayesian approach to get not just credibility estimates for risk profiles but their posterior distributions
- Modelling high frequency low impact losses (with lower threshold)
- Allocation of capital into Business Units
- Combining expert opinions and external data