The University of New South Wales
School of Mathematics and Statistics

Mathematics Drop–in Centre

POLYNOMIAL DIVISION

You need to be able to divide one polynomial by another, giving a quotient and remainder, and to write the result of your calculation as an equation. The working can be set out using long division, as in the following example.

Problem. Divide $2x^3 + 3x^2 - 4x - 9$ by $x^2 - x - 3$.

Solution. The line numbers in the following are just for explanation and you do not need to write them in your working.

$$
\begin{array}{rll}
 & 2x + 5 & (1) \\
x^2 - x - 3 \,\big) & 2x^3 + 3x^2 - 4x - 9 & (2) \\
 & 2x^3 - 2x^2 - 6x & (3) \\
 & 5x^2 + 2x - 9 & (4) \\
 & 5x^2 - 5x - 15 & (5) \\
 & 7x + 6 & (6)
\end{array}
$$

This shows that if $2x^3 + 3x^2 - 4x - 9$ is divided by $x^2 - x - 3$, the quotient is $2x + 5$ and the remainder is $7x + 6$. This can be written as the equation

$$2x^3 + 3x^2 - 4x - 9 = (x^2 - x - 3)(2x + 5) + (7x + 6) \ ,$$

and you can check your working by multiplying out the right hand side.

Explanation of the method.

- Write the given polynomials as in line (2).
- Look at the first term of each, that is, $2x^3$ and x^2; divide to get $2x$; this is the first term of the quotient, and is written in line (1), aligned with the x term beneath it.
- Next, multiply this $2x$ by the divisor $x^2 - x - 3$, giving the product $2x^3 - 2x^2 - 6x$; write it in line (3), making sure that the terms are correctly aligned with line (2).
- Subtract the $2x^3 - 2x^2 - 6x$ in line (3) from the expression $2x^3 + 3x^2 - 4x$ above it to get $5x^2 + 2x$, and bring down the next term -9 from line (2); write the total, $5x^2 + 2x - 9$ in line (4), aligned with previous lines.
- Now we treat the $5x^2 + 2x - 9$ in line (4) in the same way as we did the $2x^3 + 3x^2 - 4x - 9$ in line (2). Divide $5x^2$ by x^2 to get 5; add this to the quotient in line (1).
- Multiply the 5 by $x^2 - x - 3$ and write the result in line (5).
- Subtract line (5) from line (4) to give $7x + 6$, which is written in line (6). As there are no further terms to bring down from line (2), the process is finished.

Another example. Divide $3x^3 - x + 4$ by $x + 2$.

Solution. The working is aligned so as to allow for missing terms.

$$
\begin{array}{rl}
 & 3x^2 - 6x + 11 \\
x + 2 \,\big) & 3x^3 \qquad - x + 4 \\
 & 3x^3 + 6x^2 \\
 & - 6x^2 - x \\
 & - 6x^2 - 12x \\
 & 11x + 4 \\
 & 11x + 22 \\
 & - 18
\end{array}
$$

The quotient is $3x^2 - 6x + 11$, the remainder is -18, and we have

$$3x^3 - x + 4 = (x + 2)(3x^2 - 6x + 11) - 18 \ .$$

EXERCISES.

Please try to complete the following exercises. Remember that you **cannot** expect to understand mathematics without doing lots of practice! Please do not look at the answers before trying the questions. If you get a question wrong you should go through your working carefully, find the mistake and fix it. If there is a mistake which you cannot find, or a question which you cannot even start, please consult your tutor or the Mathematics Drop–in Centre.

1. Finding the quotient and remainder, and writing your result as an equation, divide
   (a) $x^3 - 2x^2 + 3x - 4$ by $x^2 + x + 4$;
   (b) $2x^3 - 5x^2 - 2x + 4$ by $x - 3$;
   (c) $x^4 + 2x^3 + 3x^2 + 5x + 7$ by $x^2 - x + 5$;
   (d) $-2x^4 + 11x^3 - 10x^2 + 7$ by $2x^2 - 3x + 2$;
   (e) $x^4 - x^3 + 3x^2 + 4x + 6$ by $x + 1$;
   (f) $x^3 + 2$ by $x - 3$;
   (g) $x^4 + x^3 + 5$ by $x^2 - 3x - 1$;
   (h) $x^5 + 2x^4 + 6x^2 - 3x + 4$ by $x^3 + x - 1$.
2. The divisions in this question will involve fractional coefficients. The method is exactly the same, but you will have to be very careful with the arithmetic.
   (a) Divide $x^3 + x^2 - 2x - 1$ by $2x + 3$.
   (b) Divide $x^3 - x^2 + 2$ by $3x^2 + x + 1$.
3. If a is an unspecified constant, find the quotient and remainder when $x^3 + ax^2 - 2x + 7$ is divided by $x^2 - 3x - 1$.

ANSWERS.

1. (a) $x^3 - 2x^2 + 3x - 4 = (x^2 + x + 4)(x - 3) + (2x + 8)$;
   (b) $2x^3 - 5x^2 - 2x + 4 = (x - 3)(2x^2 + x + 1) + 7$;
   (c) $x^4 + 2x^3 + 3x^2 + 5x + 7$
   $= (x^2 - x + 5)(x^2 + 3x + 1) + (-9x + 2)$;
   (d) $-2x^4 + 11x^3 - 10x^2 + 7$
   $= (2x^2 - 3x + 2)(-x^2 + 4x + 2) + (-2x + 3)$;
   (e) $x^4 - x^3 + 3x^2 + 4x + 6 = (x + 1)(x^3 - 2x^2 + 5x - 1) + 7$;
   (f) $x^3 + 2 = (x - 3)(x^2 + 3x + 9) + 29$;
   (g) $x^4 + x^3 + 5 = (x^2 - 3x - 1)(x^2 + 4x + 13) + (43x + 18)$;
   (h) $x^5 + 2x^4 + 6x^2 - 3x + 4$
   $= (x^3 + x - 1)(x^2 + 2x - 1) + (5x^2 + 3)$.
2. (a) $x^3 + x^2 - 2x - 1 = (2x + 3)(\frac{1}{2}x^2 - \frac{1}{4}x - \frac{5}{8}) + \frac{7}{8}$.
   (b) $x^3 - x^2 + 2 = (3x^2 + x + 1)(\frac{1}{3}x - \frac{4}{9}) + (\frac{1}{9}x + \frac{22}{9})$.
3. Quotient $x + (a + 3)$, remainder $(3a + 8)x + (a + 10)$.