The University of New South Wales
School of Mathematics and Statistics

Mathematics Drop–in Centre

QUADRATICS, PART 1

Many important problems in first–year (and further) mathematics have solution methods involving quadratic equations. It is essential that you are able to solve quadratics and get them right every time! A quadratic equation can be written in the form

$$ax^2 + bx + c = 0 \ ,$$

where a is a given number, which must not be zero, and b, c are given numbers which may be zero. That is, the equation *definitely* has an x^2 term, *may* have an x term, and *may* have a constant term. We call a, b and c the *coefficients* of the quadratic. There are three methods of solving a quadratic: sometimes one is easier, sometimes another, so it is recommended that you practise all three.

Solving by inspection. This method does not always work easily, but is often effective if $a = 1$ and b, c are integers; we recommend that you at least give it a try before turning to other methods. The solutions of the quadratic will be numbers α, β such that

$$\alpha + \beta = -b \quad \text{and} \quad \alpha\beta = c \ ,$$

and with a bit of practice these can often be found by trial and (not much) error. For example, to solve

$$x^2 - 8x + 15 = 0 \tag{1}$$

we seek two numbers whose product is 15 and whose sum is 8. The obvious try for two numbers with product 15 is 3 and 5, and these do indeed have sum 8, so we are done! Quadratic (1) has solutions $x = 3$ and $x = 5$. What if we had

$$x^2 + 8x + 15 = 0 \ ? \tag{2}$$

Then we should want numbers with product 15 and sum -8. Once we have thought of the possible solutions 3 and 5, which do not add up to -8, it should be easy to realise that -3 and -5 also have product 15, and they do add up to -8. So (2) has solutions $x = -3$ and $x = -5$. Another example is

$$x^2 + 2x - 15 = 0 \ . \tag{3}$$

Still thinking about 3 and 5, we notice that this time the product has to be -15, so we need either 3 and -5, or -3 and 5. As the sum of the solutions is to be -2, the former option is the correct one: the solutions are $x = 3$ and $x = -5$. If we have

$$x^2 + 14x - 15 = 0 \tag{4}$$

then playing around with 3s and 5s does not seem to help. But hopefully we realise that we could think about 15 and 1 instead, and soon we find the solutions $x = -15$ and $x = 1$. For

$$x^2 + 8x - 15 = 0 \tag{5}$$

nothing seems to work easily, and rather than spend any more time trying to solve the equation by inspection, we'll just go on to one of the other methods. See the worksheet *Quadratics, part 2* to learn about **completing the square** and **the quadratic formula**.

EXERCISES.

Please try to complete the following exercises. Remember that you **cannot** expect to understand mathematics without doing lots of practice! Please do not look at the answers before trying the questions. If you get a question wrong you should go through your working carefully, find the mistake and fix it. If there is a mistake which you cannot find, or a question which you cannot even start, please consult your tutor or the Mathematics Drop–in Centre.

1. As mentioned in the text, quadratics can sometimes be solved easily by inspection, and sometimes not. It is important to develop some kind of feeling for how long you should try before giving up and using a different method. So, for the following quadratics, *either* solve them by inspection, *or* decide that they are too hard to do by this method.
   (a) $x^2 - 13x + 22 = 0$;
   (b) $x^2 - 9x = 22$;
   (c) $x^2 + 21x - 22 = 0$;
   (d) $x^2 + x - 22 = 0$;
   (e) $x^2 + 7x - 30 = 0$;
   (f) $x^2 - 11x - 30 = 0$;
   (g) $x^2 - 11x + 30 = 0$;
   (h) $x^2 + 90x = 1000$;
   (i) $x^2 - 10x + 25 = 0$.
2. Here are three simple quadratics which for some reason often confuse students – see if you can get them right first time!
   (a) $x^2 - 5x + 6 = 0$;
   (b) $x^2 - 5x - 6 = 0$;
   (c) $x^2 + x - 6 = 0$.
3. Check your answers to questions 1 and 2 by substituting your x values into the equation and confirming that LHS = RHS.

ANSWERS.

1. (a) $x = 2$, $x = 11$;
   (b) $x = -2$, $x = 11$;
   (c) $x = 1$, $x = -22$;
   (d) too hard;
   (e) $x = 3$, $x = -10$;
   (f) too hard;
   (g) $x = 5$, $x = 6$;
   (h) $x = 10$, $x = -100$;
   (i) there is only one solution $x = 5$; alternatively we can say that the "two" solutions are both the same, $x = 5$ and $x = 5$, because $5 \times 5 = 25$ and $5 + 5 = 10$.
2. (a) $x = 2$, $x = 3$;
   (b) $x = -1$, $x = 6$;
   (c) $x = 2$, $x = -3$.