MATHEMATICS ENRICHMENT CLUB.[1]
Problem Sheet 1, April 30, 2012

1. What is the least positive integer n such that $60 \times n$ is a cube?

2. A certain substance doubles in volume every 5 minutes. At 9.00am, a small amount is placed in a container and at 10.00am, the container just fills. When was it one quarter full? How full was it at 9.30am?

3. In how many ways is it possible to write 1000 as a sum of consecutive odd integers?

4i. Show that a number is divisible by 3 if the sum of its digits is divisible by 3.

ii. Show that no 7 digit number make up from the digits 0,1,2,3,4,5,6 (no repetition) can be a square, a cube, or any higher power.

5. Let $\tau(n)$ be the number of divisors of a number n.
(i) Find $\tau(20)$ and show that τ of a square is odd.

(ii) Explain why it is impossible to have $\tau(n^2) = 2\tau(n)$.

(iii) Can you find integers n such that $\tau(n^2) = 3\tau(n)$?

6. Two figures are similar if one is a magnification of the other. That is, all their angles are the same and the corresponding sides are in proportion.

(i) What are the (minimal) conditions for two squares to be similar? - two rectangles? - two trapezia ?

(ii) Show how to divide a trapezium into two similar trapezia.

7.(i) Show that the median to the hypotenuse of a right-angled triangle has length exactly half the length of the hypotenuse.

(ii) Let A, B, C be a triangle with A_1, B_1, C_1 the midpoints of the sides BC, CA, AB re-

[1]Some of the problems here come from T. Gagen, Uni. of Syd. and from E. Szekeres , Macquarie Uni.

spectively. Let D be the foot of the perpendicular from A to BC. Show that B_1C_1D is congruent to $B_1C_1A_1$.

8. Eratosthenes (276-194 BC) observed in the town of Swenet, that at noon of the summer solstice the sun is directly overhead. He also observed that in the town of Alexandria, that at noon of the summer solsitce the sun is *almost* directly overhead. It is off by $7^o12^{'}$.

Given that Swenet and Alexandria are 5000 stadia apart (40,000km), what is the circumference of the world?

Senior Questions.

1. Find all values of x satisfying the pair of equations $x^2 - px + 20 = 0$ and $x^2 - 20x + p = 0$

2. Find the sum $S_n = \dfrac{1}{1 \times 4} + \dfrac{1}{4 \times 7} + \dfrac{1}{7 \times 10} + ... + \dfrac{1}{(3n-2) \times (3n+1)}$.

3. In triangle ABC it is known that $\sin B - \sin C = 2 \sin A$. Prove that $\tan \frac{B}{2} \cot \frac{C}{2} = -3$.