UNSW
THE UNIVERSITY OF NEW SOUTH WALES

Never Stand Still | Faculty of Science | School of Mathematics and Statistics

MATHEMATICS ENRICHMENT CLUB.[1]
Problem Sheet 4, May 21, 2012

1. If $f(n) = (n-1)f(n-1)$ and $f(1) = 1$ find $f(4)$.

2. The product of the ages in years of two adults is 770. What is the sum of their ages?

3. (a) How many positive integers are there ≤ 100 which have no factors, except 1, in common with 100?

   (b) What is their sum?

4. If $x_1 = 3$, the recurrence $x_{n+1} = x_n^2 - 10x_n$, gives the sequence $3, -21, 651, 417291...$ and the numbers increase without bound. Find all the values for x_1 so that the sequence does NOT increase without bound.

5. Solve the simultaneous equations:

$$x + yz = 2$$

$$y + xz = 2$$

$$z + xy = 2.$$

6. Two circles C_1, C_2 with centres O_1, O_2 are externally tangent at the point P. A straight line through P meets C_1, C_2 respectively at A and B. Show that the tangents to the circles at A and B are parallel.

7. Let $ABCD$ be a trapezium with $AB||CD$. Let P be the intersection of the diagonals AC and BD.

   (a) Show that the triangles APD and BPC have the same area.

   (b) Given that APB has area 1 cm^2 and that APD has area 4 cm^2, find the area of $ABCD$.

[1]Some of the problems here come from T. Gagen, Uni. of Syd. and from E. Szekeres , Macquarie Uni.

Senior Questions.

1. Find $\displaystyle\int \frac{1}{x+\sqrt{x}}\,dx$.

2. Find $\displaystyle\lim_{n\to\infty} \frac{n!}{n^n}$.

3. Prove that
$$1 \times 3 \times 5 \times ... \times (2n-1) = \frac{(2n)!}{2^n n!}.$$