UNSW
THE UNIVERSITY OF NEW SOUTH WALES

Never Stand Still | Faculty of Science | School of Mathematics and Statistics

MATHEMATICS ENRICHMENT CLUB.[1]
Problem Sheet 11, August 2, 2012

1. Solve $\dfrac{x+3y}{2x+5y} = \dfrac{4}{7}$.

2. Find a number less than 100 which is increased by 20% when the digits are reversed.

3. (a) Verify that

$$\begin{aligned} x^{15} - 1 &= (x^3-1)(x^{12}+x^9+x^6+x^3+1) \\ &= (x^5-1)(x^{10}+x^5+1). \end{aligned}$$

   (b) Hence factor $2^{15} - 1$ as a product of prime factors.

   (c) Can you factorise $2^{15} + 1$ as a product of prime factors?

4. Suppose that P is a point inside a rectangle $ABCD$ with $AB = 15$cm, and $AD = 10$cm. If $PA = 14$cm and $PB = 11$cm, find PD in surd form.

5. Find all positive integers m and n such that $3m - 1$ is a multiple of n and $3n - 1$ is a multiple of m.

   (Hint: Suppose $m \le n$, then n divides $3m - 1 < 3m \le 3n$.)

6. (a) Let M be the midpoint of the side BC of the triangle ABC and let N be the midpoint of AC. Suppose that AM and BN meet at S. Show that

$$AS : SM = BS : SN = 2 : 1.$$

   (b) Hence show that the medians of a triangle are concurrent.

7. (a) Let M be the midpoint of the side AB in the triangle ABC. If CM has length h, prove that

$$2(a^2 + b^2) = c^2 + 4h^2.$$

   This is known as *Apollonius' theorem.*

   (b) Show how to draw a triangle knowing only the lengths of the three medians h, k and ℓ. (You can either use (i), or find a better way.)

[1] Some of the problems here come from T. Gagen, Uni. of Syd. and from E. Szekeres , Macquarie Uni.