MATHEMATICS ENRICHMENT CLUB.[1]
Problem Sheet 7, June 18, 2013

1. (a) Let M be the midpoint of the side AB in the triangle ABC. If CM has length h, prove that
$$2(a^2 + b^2) = c^2 + 4h^2.$$
This is known as *Apollonius' theorem.*

   (b) Show how to draw a triangle knowing only the lengths of the three medians h, k and ℓ. (You can either use (i), or find a better way.)

2. Two circles C_1, C_2 with centres O_1, O_2 are externally tangent at the point P. A straight line through P meets C_1, C_2 respectively at A and B. Show that the tangents to the circles at A and B are parallel.

3. Find the last two digits (and then the last three digits) of $1! + 2! + 3! + ... + 99!$.

4. Denote the top of a cube by $ABCD$ and the bottom by A_1, B_1, C_1, D_1, so that A is directly above A_1 and so on. Take midpoints of the six edges $AB, BB_1, B_1C_1, C_1D_1, D_1D$ and DA. show that a plane containing any three of these points contains them all and deduce that these points form the vertices of a regular hexagon.

5. A quadrilateral in which a circle can be drawn which touches each of the four faces is called a *circumscribable quadrilateral*. If r is the radius of the circle and s is half the perimeter of the quadrilateral, prove that the area of the quadrilateral is rs.

6. What is the smaller angle between the hands of the clock at 12:25pm?

Senior Questions

1. Solve the equation $\cot^{-1} x - \cot^{-1}(x + 2) = \frac{\pi}{12}$.

2. If x is a number between 4 and 8 and y is a number between 20 and 40, what are the smallest and largest possible values of $\frac{y}{x}$?

[1]Some of the problems here come from T. Gagen, Uni. of Syd. and from E. Szekeres , Macquarie Uni.