MATHEMATICS ENRICHMENT CLUB.[1]
Problem Sheet 8, June 25, 2013

1. Find the difference between the sum of the first 100 even integers and the first 100 odd integers.

2. In how many ways can 6 boys and 6 girls stand next to each other in a row such that no two boys stand next to each other and no two girls stand next to each other?

3. In how many essentially different ways can we paint the faces of a tetrahedron using

   (a) 2 different colours?
   (b) 3 different colours?
   (c) 4 different colours?

4. Let ABC be an isosceles triangle with the base angles B and C being $72°$ and $AB = AC = 4$. The length of the base BC, called x is chosen such that a line CD can be drawn, where D lies on AB, such that $\angle BDC = 72°$.

   (a) Find a pair of similar triangles and show that x satisfies, $x^2 + 4x - 16 = 0$.
   (b) Use triangle ABC to find $\cos 72°$ in surd form.
   (c) Use triangle ACD to find $\cos 36°$ in surd form.

5. Professor Farnsworth has invented a machine that swaps the minds and bodies of two people. Amy and the Professor swap bodies, and after realising they much preferred their own bodies, tried to swap back, only to discover that each pair of bodies can only be swapped by the machine exactly once, so that the Amy-body and Professor-body cannot swap minds again.

   (a) How many extra people do Amy and the Professor need to set things right?
   (b) If n people find themselves in another's body and all want to swap back to their original bodies, how many extra bodies do they need?

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[1]Some of the problems here come from T. Gagen, Uni. of Syd. and from E. Szekeres , Macquarie Uni. Question 5 comes from the Futurama episode "The Prisoner of Benda".

Senior Questions

1. Prove that $\log_2 3$ is irrational.

2. (a) What number is one less than 11010000 in base 3?

   (b) What is the decimal representation of 220200_3 ?