

MATHEMATICS ENRICHMENT CLUB.¹

Problem Sheet 10, July 30, 2013

1. Simplify $(x^{-1} + y^{-1})^{-1}$.
2. What is the least positive integer n such that $60 \times n$ is a cube?
3. Show that the number 13950264876 is not a square by thinking about divisibility by 3.
4. The angles in a triangle are in the ratio 2 : 3 : 4. Find, in degrees, the size of the largest angle.
5. Suppose the median from the vertex C of a triangle ABC has length $\frac{1}{2}AB$. Show that the triangle is right-angled at C .
6. (a) Find the greatest common divisor of $2^{50} + 1$ and $2^{20} + 1$.
(b) Explain why the greatest common divisor of $2^m + 1$ and $2^n + 1$ is at least 3 if m and n are both odd.

Senior Questions

1. Gabriel's Horn is constructed by rotating the graph $y = \frac{1}{x}$, $x \geq 1$ about the x -axis.
 - (a) Prove that Gabriel's Horn is infinite in surface area.
 - (b) What is the volume of Gabriel's Horn?
2. Prove that
$$\cos((n+2)\theta) = 2\cos((n+1)\theta)\cos\theta - \cos(n\theta),$$
for each integer $n \geq 0$. Hence express $\cos 5\theta$ in terms of powers of $\cos \theta$.

¹Some of the problems here come from T. Gagen, Uni. of Syd. and from E. Szekeres, Macquarie Uni.