MATHEMATICS ENRICHMENT CLUB.[1]
Problem Sheet 11, August 6, 2013

1. (a) Explain why ab is greatest when $a = b = \frac{k}{2}$.
   (b) Suppose that $x^2 + y^2 = c^2$, find the minimum value of $x^4 + y^4$.

2. Let P be a point outside a circle with diameter AB and let Q be a point inside it. Prove that $\angle APB$ is acute and that $\angle AQB$ is obtuse.

3. (a) Without using a calculator, explain why the quadratic equation
$$x^2 + 2343643x - 2382987 = 0$$
has no integer solutions.
   (b) Without using a calculator, explain why the quadratic equation
$$x^2 + 2343644x - 2382982 = 0$$
has not integer solutions.

4. In how many ways can we change \$10 into 50 cent and 20 cent coins, with at least one of each coin being used.

5. Let $\tau(n)$ be the number of divisors of a number n.
   (a) Find $\tau(20)$ and show that τ of a square is odd.
   (b) Explain why it is impossible to have $\tau(n^2) = 2\tau(n)$.
   (c) Can you find integers n such that $\tau(n^2) = 3\tau(n)$?

6. Find infinitely many integers x such that
$$\sqrt[3]{x + \sqrt{x^2+1}} + \sqrt[3]{x - \sqrt{x^2+1}}$$
is an integer.

[1]Some of the problems here come from T. Gagen, Uni. of Syd. and from E. Szekeres , Macquarie Uni.

Senior Questions

1. (a) Show that $n^4 - 6n^3 - 18n^2 + 6n + 1 = (n^2 - 3n - 1)^2 - 25n^2$

   (b) Hence find all integers n such that $n^4 - 6n^3 - 18n^2 + 6n + 1$ is prime.

2. Garen and Katarina play 100 games of checkers to determine who is the superior checkers player. How many games must one of them win to be able to say that there is at least a 95% chance they didn't win all their games by luck alone?