MATHEMATICS ENRICHMENT CLUB.[1]
Problem Sheet 14, August 27, 2013

1. Suppose n is an integer greater than 1.
   (a) Show that $n^2 + n$ lies strictly between two squares and so cannot be a square.
   (b) Use the same idea to show that $n^4 + n^3 + n^2 + n$ is not a perfect square. (Note that it is when $n = 1$.)
2. What is the sum of all the digits used in writing down the numbers from one to 9999?
3. Is it possible to put an equilateral triangle onto a square grid so that all vertices are in corners?
4. What is the smallest number divisible by $1, 2, 3, \ldots, 10$?
5. 25 checkers are placed on the 25 leftmost squares of a $1 \times N$ board. A checker can either move to the empty adjacent square to its right or jump over an adjacent right checker to the next square if it is empty. Moves to the left are not allowed. Find the smallest N so that all the checkers can be rearranged to be in 25 successive squares, but in the reverse order.
6. Given an equilateral triangle ABC and a point O inside it, with $\angle BOC = x$ and $\angle AOC = y$, find, in terms of x and y, the angles of the triangle with side lengths equal to AO, BO and CO.

Senior Questions

1. It can be shown that the sum to infinity of the series
$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots.. = \frac{\pi^2}{6}.$$
   Use this to find the sum to infinity of the series
$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \ldots..$$
   and the sum to infinity of
$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots..$$

[1] Some of the problems here come from T. Gagen, Uni. of Syd. and from E. Szekeres , Macquarie Uni. Senior problem 2 provided by David Treeby. Some problems from the Tournament of Towns

2. Robin Hood's apprentice still has a lot to learn when it comes to archery. She can always hit the target, but is equally likely to hit any point on the target. On a square target, is she more likely to land an arrow closer to the bullseye (the centre) or to an edge?